18.022 Recitation Handout (with solutions)
8 September 2014

1. (1.1.25 in *Colley*) Physical forces are quantities possessing both magnitude and direction and therefore can be represented by vectors. If an object has more than one force acting on it, then the resulting force can be represented by the sum of the individual force vectors. Suppose that two forces $\mathbf{F}_1 = (2,7,-1)$ and $\mathbf{F}_2 = (3,-2,5)$ are acting on an object.

(a) What is the resultant force of $\mathbf{F}_1$ and $\mathbf{F}_2$?

Solution. The resultant force is the vector sum of $\mathbf{F}_1$ and $\mathbf{F}_2$, which is obtained by adding the components of the two vectors: $\mathbf{F}_1 + \mathbf{F}_2 = (2+3, 7-1, -1+5) = \boxed{(5,5,4)}$

(b) What force $\mathbf{F}_3$ is needed to counteract these forces, so that no net force results and the object remains at rest?

Solution. If $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$, then $\mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2) = \boxed{(-5,-5,-4)}$.

2. (a) Consider the vectors $u_1 = (1,1,0)$, $u_2 = (-1,1,0)$, and $u_3 = (0,1,1)$. Write $(3,-6,4)$ as a linear combination of u_1, u_2, and u_3. In other words, find real numbers c_1, c_2, and c_3 such that $(3,-6,4)$ is equal to $c_1u_1 + c_2u_2 + c_3u_3$.

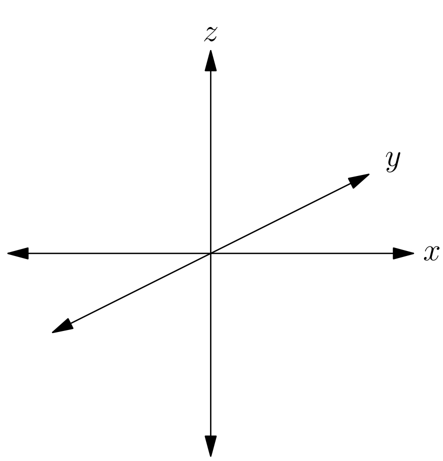
Solution. Writing out $c_1u_1 + c_2u_2 + c_3u_3 = (3,-6,4)$ coordinate-by-coordinate, we obtain the equations $c_1 - c_2 = 3$, $c_1 + c_2 = -6$, and $c_3 = 4$. Solving this system, we find $c_1 = -3/2$, $c_2 = 9/2$, and $c_3 = 4$.

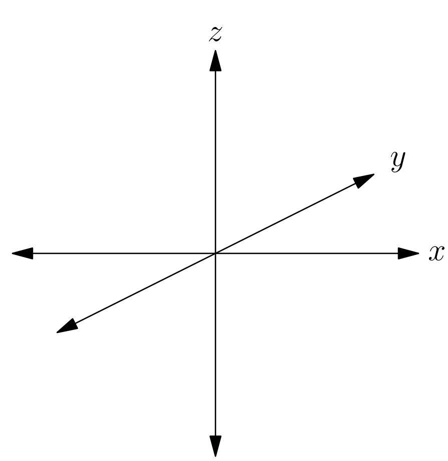
(b) Consider the vectors $v_1 = (1,1,0)$, $v_2 = (-1,1,0)$, and $v_3 = (0,-2,0)$. Can you write $(3,-6,4)$ as a linear combination of v_1, v_2, and v_3 ?

Solution. No, because it is not possible to obtain anything other than 0 in the third coordinate of a linear combination of v_1, v_2, and v_3.

(c) Sketch u_1, u_2, u_3 and v_1, v_2, v_3 on the axes below. What is the key difference between the two sets of vectors?

Solution. The key difference is that v_1, v_2, and v_3 all lie in the same plane, while u_1, u_2, u_3 do not.





3. A chickadee starts at the point $(2,-4,1)$ and flies in the direction of the vector $\left(\frac{3}{\sqrt{10}},0,\frac{1}{\sqrt{10}}\right)$ at a rate of $\sqrt{10}$ units per second. A hummingbird starts at the point $(8,20,7)$ and flies in the direction of the vector $\left(\frac{1}{\sqrt{5}},-\frac{2}{\sqrt{5}},0\right)$ at a rate of $3\sqrt{5}$ units per second.

(a) Do the paths of the chickadee and the hummingbird intersect?

Solution. Since the chickadee starts at $(2,-4,1)$ and moves at a rate $\sqrt{10}$ times the vector $(\frac{3}{\sqrt{10}},0,\frac{1}{\sqrt{10}})$, the parametric equations describing the line on which it moves are $(x,y,z)=(2+3t,-4,1+t)$. Similarly, the humming bird moves on the line $(x,y,z)=(8+s,20-2s,7)$. Setting these ordered triples equal, we obtain a solution with $s=4$ and $t=6$. Therefore, the paths do intersect.

(b) Do the hummingbird and the chickadee collide?

Solution. The birds collide if they reach the intersection point at the same time. Since the chickadee takes 6 seconds while the hummingbird takes 4 seconds, they do not collide.

(c) Suppose that the velocity of the hummingbird is k units per second. For what value of k do the hummingbird and the chickadee collide?

Solution. The hummingbird collides with the chickadee if it takes 6 seconds to reach their intersection point. By part (a) above, it actually takes 4 seconds. Therefore, it needs to slow down by a factor of $2/3$, which makes its speed equal to $\boxed{2\sqrt{5}}$.

4. Suppose that a particle is revolving clockwise around the point $(4,2)$ at a rate of 3 revolutions per second. Write parametric equations describing the location of the particle at time t, assuming that it starts at the point $(6,2)$.

Solution. We begin with the parametric equations $(\cos t,\sin t)$ for the unit circle. To scale by a factor of 2, we multiply the x-coordinate and the y-coordinate by a factor of 2. We then translate 4 units right and 2 units up by adding 4 to the x-coordinate and 2 to the y-coordinate. Finally, we replace t with $-6\pi t$ to account for the reversal of direction as well as the increase in speed. We choose the factor 6π since we want to make 3 revolutions by time $t=1$. Altogether, we get $(4+2\cos(-6\pi t),2+2\sin(-6\pi t))=$ $\boxed{(4+2\cos(6\pi t),2-2\sin(6\pi t))}$