18.022 Recitation Handout
03 December 2014

1. Find the flux of the vector field $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j}$ across the surface

$$z = 1 - x^2 - y^2, \quad z \geq 0.$$

2. Write the surface $\left(s + t, s^2 + t^2, 3st(s + t)\right)$ (for $(s, t) \in \mathbb{R}^2$) as the graph of a function $f(x, y)$.

3. Calculate $\int_{\partial D} xy\,dS$, where $D = [0,1]^3$.

4. (Fun/Challenge problem, 7.2.25 in *Colley*) Let a be some positive constant. Consider the surface defined by $\mathbf{X}(s,t) = (x(s,t), y(x,t), z(s,t))$, where

$$\begin{aligned} x(s,t) &= \left(a + \cos\frac{s}{2}\sin t - \sin\frac{s}{2}\sin 2t\right)\cos s, \\ y(s,t) &= \left(a + \cos\frac{s}{2}\sin t - \sin\frac{s}{2}\sin 2t\right)\sin s, \\ z(s,t) &= \sin\frac{s}{2}\sin t + \cos\frac{s}{2}\sin 2t, \end{aligned}$$

and s and t each vary over $[0, 2\pi]$.

(a) Describe the s-coordinate curve at $t = 0$.

(b) Calculate the standard normal vector $\mathbf{N}$ along the s-coordinate curve at $t = 0$. In other words, find $\mathbf{N}(s, 0)$.

(c) Note that $\mathbf{X}(0,0) = \mathbf{X}(2\pi, 0)$. Compare $\mathbf{N}(0,0)$ and $\mathbf{N}(2\pi, 0)$. What can you conclude about the surface?