18.022 Recitation Handout
24 November 2014

1. According to Coulomb's law, the force between a particle of charge q_1 at the origin and a particle of charge q_2 at the point $\mathbf{r} = (x, y, z) \in \mathbb{R}^3$ is given by

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3},$$

where ε_0 is a physical constant.

(a) Is $\mathbf{F}$ a conservative vector field? If so, find a function $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla\phi = \mathbf{F}$.

(b) If the distance between two charges is tripled, by what factor is the force between them reduced?

(c) How much work is required to move the second particle along the path

$$\gamma(t) = (1 + (1 - t)\cos(t^2),\ \sqrt{\sin \pi t}, 4t - t^2) \qquad 0 \le t \le 1?$$

Express your answer in terms of q_1, q_2, and ε_0.

2. (6.2.23 in *Colley*) Let D be a region to which Green's theorem applies and suppose that $u(x, y)$ and $v(x, y)$ are two functions of class C^2 whose domains include D. Show that

$$\iint_D \frac{\partial(u, v)}{\partial(x, y)}\, dA = \oint_C (u\nabla v) \cdot d\mathbf{s},$$

where $C = \partial D$ is oriented as in Green's theorem.

3. (6.1.29 in *Colley*) Let C be a level set of the function $f(x, y)$. Show that $\int_C \nabla f \cdot d\mathbf{s} = 0$.