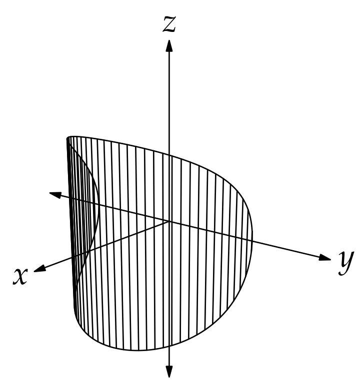
18.022 Recitation Handout
17 November 2014

1. Consider the surface $S = \left\{(x, y, z) \in \mathbb{R}^3 : x > 0 \text{ and } r = 1 \text{ and } -\sqrt{\frac{\pi^2}{4} - \theta^2} \leq z \leq \sqrt{\frac{\pi^2}{4} - \theta^2}\right\}$, shown below. (Note that r and θ refer to cylindrical coordinates.)

(a) Find the surface area of S using a scalar line integral.

(b) Check your answer by finding a non-calculus method of calculating the area of S.



2. In this problem, we discover a curl-free vector field which is not conservative.

(a) Define the vector field $\mathbf{F}(x, y, z) = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right)$. Show that $\nabla \times \mathbf{F} = \mathbf{0}$.

(b) Show that the line integral of $\mathbf{F}$ around the origin-centered unit circle in the x-y plane does not vanish.

(c) How do you reconcile parts (a) and (b)?