

18.022 Recitation Handout  
3 November 2014

1. Evaluate  $\int_0^1 \int_0^{y^2} x^2 y \, dx \, dy$  and sketch the region of integration in  $\mathbb{R}^2$  indicated by the limits of integration.

2. Evaluate  $\int_0^\pi \int_y^\pi \frac{\sin x}{x} \, dx \, dy$ .

3. (Putnam exam '89) Evaluate  $\int_0^a \int_0^b e^{\max\{b^2 x^2, a^2 y^2\}} dy dx$  where  $a$  and  $b$  are positive.

4. (Fun/Challenge, based on 5.2.29 in *Colley*) Define a function  $f(x, y)$  on  $[0, 1] \times [0, 2]$  by

$$f(x, y) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational and } y \leq 1 \\ 2 & \text{if } x \text{ is irrational and } y > 1. \end{cases}$$

Show that the iterated Riemann integral  $\int_0^1 \int_0^2 f(x, y) dy dx$  exists, and find its value. Show that the iterated Riemann integral  $\int_0^2 \int_0^1 f(x, y) dx dy$  does not exist.