

18.022 Recitation Handout
20 October 2014

1. (3.2.17 in *Colley*) Use the formula

$$\kappa = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$$

to show that if f is C^2 on an interval $[a, b]$ then the curvature of the graph $y = f(x)$ is

$$\kappa = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}.$$

2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a map defined by $f(\mathbf{x}) = (|\mathbf{x}|^2, 1, |\mathbf{x}|)$ for $\mathbf{x} \in \mathbb{R}^2$. Find the total derivative Df .

3. Sketch the curve $\mathbf{x}(t) = (t \cos t, t \sin t)$ and find its unit tangent vector.

4. Let $f(x, y) = \log(x^2 + y^2)$ for $(x, y) \in \mathbb{R}^2 \setminus \{(0, 0)\}$. Show that $\nabla \cdot (\nabla f) = 0$.