18.022 Recitation Handout
15 October 2014

1. Sketch the image of the path $\mathbf{x}(t) = (\cos t, \sin 2t)$.

2. Find the arclength of the graph of $f(x) = \frac{2}{3}(x-1)^{3/2}$ between the points $(1,0)$ and $(4, 2\sqrt{3})$.

3. Consider the function $\mathbf{F}: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $\mathbf{F}(x, y, z) = (3x/y, 2x + e^z)$.

(a) Find $D\mathbf{F}$.

(b) Show that there exists an open set $U \subset \mathbb{R}$ containing 1 and a function $\mathbf{f}: U \to \mathbb{R}^2$ such that for all $x \in U$, the equations $\mathbf{F}(x, y, z) = \mathbf{F}(1, -2, 0)$ have a unique solution $(y, z) = \mathbf{f}(x)$. Show that $\mathbf{f}$ is C^1.

(c) Find $D\mathbf{f}(1)$.

4. (from 3.2.15 in *Colley*) Determine the moving frame $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$, the curvature, the torsion, and the arc length parameter $s(t)$ for the curve

$$\mathbf{x} = \left(5, \frac{1}{3}(t+1)^{3/2}, \frac{1}{3}(1-t)^{3/2}\right), \qquad -1 < t < 1.$$