18.022 Recitation Handout
6 October 2014

1. Let $f(x,y) = e^{3x+y}$, and suppose that $x = s^2 + t^2$ and $y = 2 + t$. Find $\partial f/\partial s$ and $\partial f/\partial t$ by substitution and by means of the chain rule. Verify that the results are the same for the two methods.

2. A conical ice sculpture melts in such a way that its height decreases at a rate of 0.001 meters per second and its radius decreases at a rate of 0.002 meters per second. At what rate is the volume of the sculpture decreasing when its height reaches 3 meters, assuming that its radius is 2 meters at that time? Express your answer in terms of π and in units of cubic meters per second.

3. Given a nonzero vector $\mathbf{a} \in \mathbb{R}^n$, what unit vector $\mathbf{u} \in \mathbb{R}^n$ maximizes the dot product $\mathbf{a} \cdot \mathbf{u}$? What unit vector *minimizes* the dot product? Prove that these really are the maximum and minimum, and comment on how this observation relates to the gradient ∇f of a function $f : \mathbb{R}^n \to \mathbb{R}$.

4. Consider the sphere S passing through the point $P = (1,2,3)$ and centered at the origin. Find the equation of the plane tangent to S at P.

5. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$. Is it possible for $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ to exist at $(0,0)$ while f is not differentiable at $(0,0)$? Prove that it isn't possible, or provide an example to show that it is possible.