18.022 Recitation Handout
1 October 2014

1. Find the equation for the plane tangent to the graph of $z = 2x + 1 + y$ at $(x, y) = (3, 2)$. Express your answer in standard form.

2. Let $f(x) = (x^3 - 8)/(x - 2)$ when $x \neq 2$ and let $f(2) = c_1$. Determine the value of the constant $c_1$ for which $f$ is continuous. Do the same for

$$g(x,y) = \begin{cases} \frac{3|x|^3 + 3|y|^3 - x^{10}\arctan(1+y)}{|x|^3 + |y|^3} & \text{if } (x,y) \neq (0,0) \\ c_2 & \text{if } (x,y) = (0,0). \end{cases}$$

3. Consider an arbitrary function $f:\mathbb{R}^2 \to \mathbb{R}$. (a) Does the value of $f(0,0)$ affect the limit at $(0,0)$? In other words, can we change the existence or value of $\lim_{(x,y)\to(0,0)} f(x,y)$ by changing the value of $f(0,0)$? (b) Consider some point $(x_0,y_0) \neq (0,0)$ which is near $(0,0)$. Does the value of $f(x_0,y_0)$ affect the limit at $(0,0)$?

4. (a) What are the level curves of the function $f(x,y) = x^2+y^2$? (b) List one vector which is tangent to the level curve through $(3,4)$. (c) Find the gradient of $f$ at $(3,4)$, and verify that the gradient is perpendicular to the tangent line from part (b). (d) Try to state a generalization of this fact to level surfaces of arbitrary functions $f:\mathbb{R}^n \to \mathbb{R}$. (e) Use part (d) to provide a different way of finding the formula for the equation of a plane tangent to the graph of $z = f(x,y)$ at a given point $P$.