18.022 Recitation Handout
29 September 2014

1. Evaluate the limit $\lim_{(x,y,z)\to(0,0,0)} \frac{2xy^2+xy}{x^2+y^2}$, or explain why the limit fails to exist. Recall that the limit exists and equals $L$ if for all $\epsilon > 0$ there exists $\delta > 0$ which may depend on $\epsilon$ such that whenever $0 < \|(x, y, z)\| < \delta$, the value of $\frac{2xy^2+xy}{x^2+y^2}$ is within $\epsilon$ of $L$. If the limit exists, find some $\delta$ that satisfies this definition. If the limit does not exist, find a value of $\epsilon$ for which no value of $\delta$ and $L$ satisfy the definition.

2. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$ for $f(x, y, z) = \frac{1}{xy} + \log(xyz) + e^x \sin(yz)$.

3. Define $f(x) = (x^2-4)/(x-2)$ when $x \neq 2$ and $f(2) = c_1$. Determine the value of the constant $c_1$ for which $f$ is continuous. Do the same for

$$g(x,y) = \begin{cases} \frac{3|x|^3+3|y|^3-x^{10}\arctan(y)}{|x|^3+|y|^3} & \text{if}(x,y) \neq (0,0) \\ c_2 & \text{if}(x,y) = (0,0). \end{cases}$$

4. Suppose that $f, g$, and $h$ are real-valued functions on $\mathbb{R}^n$, and suppose that $f(\mathbf{x}) = g(\mathbf{x})h(\mathbf{x})$. Prove that if $g$ is bounded and $\lim_{\mathbf{x}\to\mathbf{0}} h(\mathbf{x}) = 0$, then $\lim_{\mathbf{x}\to\mathbf{0}} f(\mathbf{x}) = 0$.