18.022 Recitation Handout
22 September 2014

1. (1.9.22 in *Colley*) Given an arbitrary tetrahedron, associate to each of its four triangular faces a vector outwardly normal to that face with length equal to the area of the face. Show that the sum of these four vectors is zero.

2. (1.9.14 in *Colley*) The median of a triangle is the line segment that joins a vertex of a triangle to the midpoint of the opposite side. The purpose of this problem is to use vectors to show that the medians of a triangle all meet at a point.

(a) Let M_1 be the midpoint of BC, let M_2 be the midpoint of AC, and let M_3 be the midpoint of AB. Write $\overrightarrow{BM_2}$ and $\overrightarrow{CM_3}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Use the fact that $\overrightarrow{CB} = \overrightarrow{CP} + \overrightarrow{PB} = \overrightarrow{CA} + \overrightarrow{AB}$ to show that P must lie two-thirds of the way from B to M_2 and two-thirds of the way from C to M_3.

(c) Use part (b) to show why all three medians must meet at P.

3. Find the equation of a plane P perpendicular to $(1,2,-1)$ containing the line that passes through the two points $(-2,5,4)$ and $(5,1,3)$. Find the distance from P to the plane P' whose equation is $x+2y-z=28$.

4. (Fun/Challenge problem) Example 9 of Section 1.5 in the book asks us to compute the distance between the lines $\ell_1(t) = (0,5,-1) + t(2,1,3)$ and $\ell_2(t) = (-1,2,0) + t(1,-1,0)$. The solution given in the book uses vectors; our goal here is to take an algebraic approach for comparison. Define $D(s,t) = |\ell_1(t) - \ell_2(t)|^2$, and find the values of s and t which minimize $D(s,t)$ (using calculus methods or otherwise).