18.022 Recitation Handout
17 September 2014

1. Determine whether $(3,1,2)$, $(-1,0,1)$, $(4,1,-2)$ is a right-handed system of vectors.

2. (1.5.32 in *Colley*) Suppose that $\ell_1(t) = t\mathbf{a}+\mathbf{b}_1$ and $\ell_2(t) = t\mathbf{a}+\mathbf{b}_2$ are parallel lines in $\mathbb{R}^2$ or $\mathbb{R}^3$. Show that the distance D between them is given by

$$D = \frac{|\mathbf{a}\times(\mathbf{b}_2-\mathbf{b}_1)|}{|\mathbf{a}|}.$$

3. (1.9.15 in *Colley*) Suppose that the four vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ in $\mathbb{R}^3$ are coplanar. Show that $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = \mathbf{0}$.

4. Show that the distance d between two parallel planes determined by the equations $Ax+By+Cz = D_1$ and $Ax+By+Cz = D_2$ is

$$d = \frac{|D_1-D_2|}{\sqrt{A^2+B^2+C^2}}.$$

5. (1.9.8 in *Colley*) Let $\mathbf{a}$ and $\mathbf{b}$ be unit vectors in $\mathbb{R}^3$. Show that

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a} \cdot \mathbf{b})^2 = 1.$$

6. (1.9.9 in *Colley*) (a) Does $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ imply $\mathbf{b} = \mathbf{c}$?

(b) Does $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ imply $\mathbf{b} = \mathbf{c}$?

7. Consider the (filled) cylinder of radius 2 and height 6 with axis of symmetry along the z-axis. Cut the cylinder in half along the y-z plane and keep one of the two resulting parts. Describe your region in cylindrical and spherical coordinates. (You have some flexibility in interpreting this question).

8. (1.9.13 in *Colley*) Mark each of the following statements with 1 if you agree, -1 if you disagree. (a) Red is my favorite color, (b) I consider myself to be a good athlete. (c) I like cats more than dogs, (d) I enjoy spicy foods, (e) Mathematics is my favorite subject. Your responses may be considered as a vector in $\mathbb{R}^5$. Suppose that you and a friend calculate your respective response vectors for this questionnaire. Explain the significance of the dot product of your two vectors.

9. (Fun/Challenge problem) The cross product on $\mathbb{R}^n$ is an anti-commutative bilinear map $\times : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ with the property that for all $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$,

$$\mathbf{x} \cdot (\mathbf{x} \times \mathbf{y}) = (\mathbf{x} \times \mathbf{y}) \cdot \mathbf{y} = 0, \text{ and} \tag{1}$$

$$|\mathbf{x} \times \mathbf{y}|^2 = |\mathbf{x}|^2 |\mathbf{y}|^2 - (\mathbf{x} \cdot \mathbf{y})^2. \tag{2}$$

(a) In words, what does (1) say?

(b) In words, what does (2) say?

(c) Prove that there is no cross product in $\mathbb{R}^2$.

(d) Show that the triple cross product identity

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{a}) = (\mathbf{a} \cdot \mathbf{a})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{a}$$

follows from (1) and (2).

(e) Use (d) to show that no nontrivial cross product exists in $\mathbf{R}^4$.

(f) Guess which integers $n > 1$ have the property that there exists a cross product in $\mathbf{R}^n$. (I don't recommend trying to work this out. Just speculate.)