18.022 Recitation Handout
10 September 2014

1. For each of the following pairs of vectors $\mathbf{a}$ and $\mathbf{b}$, calculate $\mathbf{a}\cdot\mathbf{b}$ and $\|\mathbf{a}\|\|\mathbf{b}\|$.

(a) $\mathbf{a} = (1,5)$ and $\mathbf{b} = (-2,3)$

(b) $\mathbf{a} = (3,-5)$ and $\mathbf{b} = (2,0)$

(c) $\mathbf{a} = (-2,4,1)$ and $\mathbf{b} = (4,1,2)$

(d) Conjecture an inequality relating $|\mathbf{a}\cdot\mathbf{b}|$ and $\|\mathbf{a}\|\|\mathbf{b}\|$ for $\mathbf{a},\mathbf{b}\in\mathbb{R}^n$.

(e) (Fun/Challenge problem) To prove the inequality conjectured in (d) (called the *Cauchy-Schwarz inequality*), expand the left-hand side of the inequality $\|\mathbf{a}+\lambda\mathbf{b}\|^2 \geq 0$, where λ is any real number.

2. (1.3.20 in *Colley*) Suppose that a force $\mathbf{F} = (1,-2)$ is acting on an object moving parallel to the vector $(4,1)$. Decompose $\mathbf{F}$ into a sum of vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1$ points along the direction of motion and $\mathbf{F}_2$ is perpendicular to the direction of motion.

3. (1.3.17 in *Colley*) Is it ever the case that the projection of $\mathbf{a}$ onto $\mathbf{b}$ and the projection of $\mathbf{b}$ onto $\mathbf{a}$ are the same vector? If so, under what conditions?

4. (1.3.25 in *Colley*) Use vectors to show that the diagonals of a parallelogram have the same length if and only if the parallelogram is a rectangle. (Hint: let **a** and **b** be vectors along two sides of the parallelogram, and express vectors running along the diagonals in terms of **a** and **b**.)

5. (1.3.23 in *Colley*) Let A, B, and C denote the vertices of a triangle. Let $0 < r < 1$. If P_1 is the point on $\overline{AB}$ located r times the distance from A to B and P_2 is the point on $\overline{AC}$ located r times the distance from A to C, use vectors to show that $\overline{P_1P_2}$ is parallel to $\overline{BC}$ and has r times the length of $\overline{BC}$.

6. (1975 USAMO) Let A, B, C, and D be four points in $\mathbb{R}^3$. Use vectors to show that

$$AB^2 + BC^2 + CD^2 + DA^2 \geq AC^2 + BD^2.$$

(This generalizes the fact that the sum of the squares of the sides of a quadrilateral is at least the sum of the squares of its diagonals.) Make a statement about when equality holds.