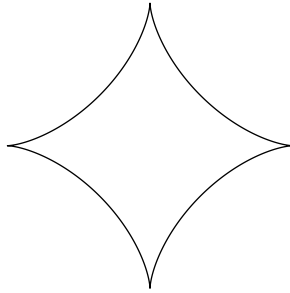


18.022 Recitation Quiz (with solutions)
26 November 2014

1. (6.2.12 in *Colley*) Use Green's theorem to find the area enclosed by the hypocycloid $\mathbf{x}(t) = (a \cos^3 t, a \sin^3 t)$, $0 \leq t \leq 2\pi$. (We found the length of the hypocycloid on 15 October). Feel free to leave an unevaluated definite integral in your answer.



Solution. Green's theorem implies that the area of a region D is given by $\oint_{\partial D} x \, dy$. We calculate

$$\begin{aligned} \oint_{\partial D} x \, dy &= \int_0^{2\pi} (a \cos^3 t)(3a \sin^2 t \cos t) \, dt \\ &= 3a^2 \int_0^{2\pi} \cos^4 t \sin^2 t \, dt = \boxed{3\pi a^2/8}. \end{aligned}$$

□