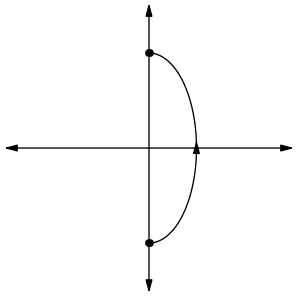
18.022 Recitation Quiz (with solutions)
19 November 2014

1. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x, y, z) = x^2y + 2y + \sqrt{z} + 3$.

(a) Find $\nabla \times (\nabla f)$.

(b) Calculate $\int_C \nabla f \cdot d\mathbf{s}$, where C is the right half of the ellipse $2x^2 + y^2 = 1$ in the x-y plane, oriented counterclockwise as shown below.



Solution. (a) The answer is $\boxed{0}$, because the curl of a gradient always vanishes.

(b) The answer is $f((0, 1, 0)) - f((0, -1, 0)) = \boxed{4}$, because $\int_a^b \nabla f \cdot d\mathbf{s} = f(b) - f(a)$. □