

18.022 Recitation Quiz (with solutions)
17 November 2014

1. Consider the two parametrizations of the line segment S from $(0, 0)$ to $(3, 3)$ given by

$$\begin{aligned} \mathbf{x}_1(t) &= (t, t) & 0 \leq t \leq 3 \\ \mathbf{x}_2(t) &= (2t, 2t) & 0 \leq t \leq 3/2. \end{aligned}$$

(a) Evaluate the line integral $\int_S x^2 dx + y dy$ using the parametrization \mathbf{x}_1 .

(b) Evaluate the line integral $\int_S x^2 dx + y dy$ using the parametrization \mathbf{x}_2 .

Solution. Using \mathbf{x}_1 , we get

$$\int_0^3 t^2 dt + t dt = \left[t^3/3 + t^2/2 \right]_0^3 = \boxed{27/2}$$

Using \mathbf{x}_2 , we get

$$\int_0^{3/2} (2t)^2 2dt + (2t) 2dt = \left[4t^3/3 + t^2 \right]_0^{3/2} = \boxed{27/2}.$$

This result is consistent with the parametrization-independence of the line integral. □