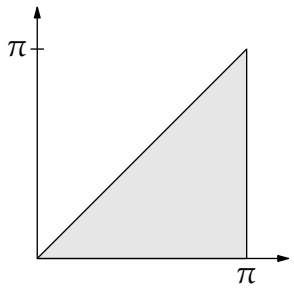
18.022 Recitation Quiz (with solutions)
3 November 2014

1. Evaluate $\int_0^\pi \int_0^x \cos(x+y)\,dy\,dx$ and sketch the region of integration in $\mathbb{R}^2$ indicated by the limits of integration.

Solution. The domain of integration is shown below.



Evaluating the integral, we find

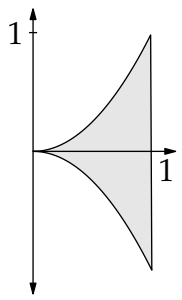
$$\begin{aligned}\int_0^\pi \int_0^y \cos(x+y)\,dy\,dx &= \int_0^\pi \left[\sin(x+y)\right]_0^x\,dx \\ &= \int_0^\pi \sin(2x)-\sin(x)\,dx \\ &= \boxed{-2}.\end{aligned}$$

2. Let $f:\mathbb{R}^2\to\mathbb{R}$ be a continuous function, and consider the integral $\int_0^1 \int_{-x^2}^{x^2} f(x,y)\,dy\,dx$.

(a) Sketch the region of integration.

(b) Rewrite the integral with the order of integration switched.

Solution. (a) The region of integration is shown below.



(b) Looking at the region in part (a) to change the order of integration, we get

$$\int_0^1 \int_{-x^2}^{x^2} f(x,y)\,dy\,dx = \int_{-1}^1 \int_{\sqrt{|y|}}^1 f(x,y)\,dx\,dy.$$