

18.022 Recitation Quiz (with solutions)
27 October 2014

1. Find the critical points of $f(x, y) = x^2 + 2xy$. Use the second derivative test for local extrema to determine whether the point is a local maximum, a local minimum, or a saddle point.

Solution. The gradient of f is $(2x+2y, 2x)$, which equals $\mathbf{0}$ if and only if $(x, y) = (0, 0)$. Therefore, the origin is the only critical point of f . The Hessian evaluated at $(0, 0)$ is

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} = 2 \cdot 0 - 2 \cdot 2 < 0,$$

so the origin is a saddle point.