18.022 Recitation Quiz (with solutions)
22 September 2014

1. Suppose that $A = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. We regard A and B as maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ by matrix multiplication (on the left, so A evaluated at $(1,2)$ is $(4,6)$, for example), and we denote by C the unit circle centered at the origin.

(a) Describe the image of C under the map AB.

Solution. The matrix B rotates the plane 90 degrees counterclockwise. Therefore, the image of C under B is the unit circle rotated 90 degrees, which is equal to C. The matrix A stretches the plane by a factor of 4 in the x direction and a factor of 3 in the y direction. Therefore, the image of C under AB is an ellipse centered at the origin with major axis of length 8 in the x direction and minor axis of length 6 in the y direction.

(b) Describe the image of C under the map BA.

Solution. If we apply A first, then we get the ellipse described in the previous question. Rotating the ellipse 90 degrees counterclockwise gives the ellipse with major axis of length 8 in the y direction and minor axis of length 6 in the x direction.