BROWN UNIVERSITY

Probability Math 1610
Lead instructor: Samuel S. Watson
Problem Set 8
Due: 19 November 2015 at 11:59 PM

Problem numbers refer to Grinstead & Snell.

1. Copy-paste the code available on the course website under Julia notebooks > Numerical Convolution to put the Geometric distribution, Poisson distribution, and the fair die distribution in order from fastest to slowest convergence to the normal distribution in the statement of the central limit theorem. You'll want to use the function `compare` at the end of that notebook–you can change the second and third arguments to `m` and `n` to see plots of all the k-fold convolutions[1] where k ranges from `m` to `n`. Tip: make sure you select Julia 0.4.0 when you open your notebook.

2. (#4 on p. 362) Suppose we choose independently 25 numbers at random from the interval $[0, 20]$ with uniform density. Write the normal densities that approximate the densities of their sum S_{25}, their standardized sum S_{25}^*, and their average A_{25}.

3. (#10 on p. 363) A surveyor is measuring the height of a cliff known to be about 1000 feet. He assumes his instrument is properly calibrated and that his measurement errors are independent, with mean $\mu = 0$ and variance $\sigma^2 = 10$. He plans to take n measurements and form the average. Estimate, using (a) Chebyshev's inequality and (b) the normal approximation, how large n should be if he wants to be 95 percent sure that his average falls within 1 foot of the true value. Now estimate, using (a) and (b), what value should σ^2 have if he wants to make only 10 measurements with the same confidence?

4. (#14 on p. 363) Physicists say that particles in a long tube are constantly moving back and forth along the tube, each with a velocity V_k (in cm/sec) at any given moment that is normally distributed, with mean $\mu = 0$ and variance $\sigma^2 = 1$. Suppose there are 10^{20} particles in the tube. (a) Find the mean and variance of the average velocity of the particles. (b) What is the probability that the average velocity is at least 10^{-9} cm/sec?

5. (from #1 on p. 374) Find the moment generating function of (a) the uniform distribution on $\{n, n+1, \ldots, n+k\}$, (b) the law of the constant random variable 3, and (c) the distribution describing a fair coin (let heads be 1 and tails -1). For each, differentiate the mgf to find the mean and variance of the distribution.

6. (#4 on p. 374) Show that if P is a probability measure supported on $\{0, 1, 2\}$, then P is determined by its first two moments μ_1 and μ_2.

7. Show that there exists a random variable X such that the moment generating function $g(t) = E(e^{tX})$ is infinite for all values of t except $t = 0$. Hint: define the law of X to be supported on $\mathbb{Z}$ and to decay slowly enough at ∞ and $-\infty$ that some of its moments are infinite.

8. Show that there exists a random variable X such that the moment generating function $g(t) = E(e^{tX})$ is infinite for all values of t such that $t > 1$ and finite for all values of t which are less than 1. Hint: define the law of X to be supported on the positive integers and to decay at just the right rate to make $E(e^X)$ infinite.

[1] The k-fold convolution is the law of the sum of k independent copies of a random variable with the given distribution.