BROWN UNIVERSITY

Probability Math 1610
Lead instructor: Samuel S. Watson
Problem Set 4
Due: 22 October 2015

Problem numbers refer to Grinstead & Snell.

Recommended problems (not required): p. 197ff: 7, 12, 17, p. 219ff: 1, 2, 38, p. 247ff: 2, 12.

1. (#31 on p. 201) In one of the first studies of the Poisson distribution, von Bortkiewicz considered the frequency of deaths from kicks in the Prussian army corps. From the study of 14 corps over a 20-year period, he obtained the data shown in the table below. Find a value of λ so that the data are well-explained by a Poisson distribution with parameter λ. Comment on why the Poisson distribution might be expected to fit the data reasonably well.

number n of deaths	number of corps with n deaths
0	144
1	91
2	32
3	11
4	2

Programming tip: you can calculate the probability mass function for the Poisson distribution with parameter `lambda` using the code

```
[exp(-lambda)*lambda^k/factorial(k) for k=0:4]
```

2. Let X and Y be independent Poisson random variables with parameters λ_1 and λ_2, respectively. Show that $X + Y$ is Poisson distributed with parameter $\lambda_1 + \lambda_2$.

3. (#5 on p. 200) Reese Prosser never puts money in a 10-cent parking meter in Hanover. He assumes that each time there is a probability of 0.05 that he will be caught. The first offense costs nothing, the second costs 2 dollars, and subsequent offenses cost 5 dollars each. Under his assumptions, how does the expected cost of parking 100 times without paying the meter compare with the cost of paying the meter each time?

4. (#28 on p. 201) An airline finds that 4 percent of the passengers that make reservations on a particular flight will not show up. Consequently, their policy is to sell 100 reserved seats on a plane that has only 98 seats. Find the probability that every person who shows up for the flight will find a seat available.

5. (#5 on p. 219) Let U be uniformly distributed in the interval $[0, 1]$, and find the cdf of the random variable $Y = |U - 1/2|$.

6. (#10 on p. 220) Find the cdfs and pdfs of $\max(U, V)$ and $\min(U, V)$ where U and V are independent random variables each of which has law $\text{Unif}([0, 1])$.

7. Let X and Y be independent $\text{Exp}(1)$ random variables. Find the cdf and pdf of $X + Y$, and find the mode of $X + Y$. (Note: the mode of a random variable whose law has pdf f is defined to be the value of x which maximizes $f(x)$.)

8. Identify the inflection points of the graph of $\mathcal{N}(\mu, \sigma^2)$ density function. (Hint: first find the inflection points for $\mathcal{N}(0, 1)$, and then apply an appropriate scaling and shift.)

9. (#6 on p. 247) A die is rolled twice. Let X denote the sum of the two numbers that turn up, and Y the difference of the numbers (first roll minus second). Show that $E(XY) = E(X)E(Y)$ but that X and Y are not independent.

10. (#15 on p. 249) A box contains two gold balls and three silver balls. You are allowed to choose successively balls from the box at random. You win 1 dollar each time you draw a gold ball and lose 1 dollar each time you draw a silver ball. After a draw, the ball is not replaced. Show that, if you draw until you are ahead by 1 dollar or until there are no more gold balls, this is a favorable game.