BROWN UNIVERSITY

Probability Math 1610
Lead instructor: Samuel S. Watson
Due: 22 September 2015

Problem numbers refer to Grinstead & Snell.

1. (#2,#3 on p. 35) Give a possible sample space Ω for each of the following experiments:

(a) An election decides between two candidates A and B.

(b) A two-sided coin is tossed.

(c) A student is asked for the month of the year and the day of the week on which her birthday falls.

(d) A student is chosen at random from a class of ten students.

(e) You receive a grade in this course.

For which of (a) through (e) would it be reasonable to assign the uniform distribution function?

2. (#7 on p. 35) Let A and B be events such that $P(A \cap B) = 1/4$, $P(\tilde{A}) = 1/3$, and $P(B) = 1/2$. What is $P(A \cup B)$? (Hint: consider Theorem 1.4)

3. (#9 on p. 36) A student must choose exactly two out of three electives: art, French, and mathematics. He chooses art with probability 5/8, French with probability 5/8, and art and French together with probability 1/4. What is the probability that he chooses mathematics? What is the probability that he chooses either art or French?

4. (#14 on p. 36) Let X be a random variable with distribution function $m_X(x)$ defined by $m_X(-1) = 1/5$, $m_X(0) = 1/5$, $m_X(1) = 2/5$, $m_X(2) = 1/5$. (a) Let Y be the random variable defined by the equation $Y = X + 3$. Find the distribution function $m_Y(y)$ of Y. (b) Let Z be the random variable defined by the equation $Z = X^2$. Find the distribution function $m_Z(z)$ of Z.

5. (#26 on p. 39) Two cards are drawn successively from a deck of 52 cards. Find the probability that the second card is higher in rank than the first card.

6. (#2 on p. 71) Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f(x) = Cx,$$

where C is a constant.

(a) Find C.

(b) Find $P(E)$, where $E = [a, b] \subset [2, 10]$. Express your answer in terms of a and b.

(c) Find $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.

7. (#6 on p. 72) Suppose that a new light bulb will burn out after t hours, where t is chosen from $[0, \infty)$ with density $f(t) = \lambda e^{-\lambda t}$ (called an *exponential* density). The parameter λ is often called the *failure rate* of the bulb.

(a) Assume that $\lambda = 0.01$, and find the probability that the bulb will not burn out before T hours. This probability is often called the *reliability* of the bulb.

(b) For what T is the reliability of the bulb equal to $1/2$?

8. (#8 on p. 72) Choose independently two numbers B and C at random from the interval [0, 1] with uniform density. Note that the point (B, C) is then chosen at random in the unit square. Find the probability that

(a) $B + C < 1/2$.

(b) $BC < 12$.

(c) $|B - C| < 1/2$.

(d) $\max\{B, C\} < 1/2$.

(e) $\min\{B, C\} < 1/2$.

(f) $B < 1/2$ and $1 - C < 1/2$.

(g) conditions (c) and (f) both hold.

(h) $B^2 + C^2 \leq 1/2$.

(i) $(B - 1/2)^2 + (C - 1/2)^2 < 1/4$.

9. (#14 on p. 73) Choose independently two numbers B and C at random from the interval $[-1, 1]$ with uniform distribution, and consider the quadratic equation $x^2 + Bx + C = 0$. Find the probability that the roots of this equation (a) are both real. (b) are both positive. Hints: (a) requires $0 \leq B^2 - 4C$, (b) requires $0 \leq B^2 - 4C$, $B \leq 0$, $0 \leq C$.

10. (#15 on p. 73) At the Tunbridge World's Fair, a coin toss game works as follows. Quarters are tossed onto a checkerboard. The management keeps all the quarters, but for each quarter landing entirely within one square of the checkerboard the management pays a dollar. Assume that the edge of each square is twice the diameter of a quarter, and that the outcomes are described by coordinates chosen uniformly at random. Is this a fair game?

11. (#23 on p. 74) Write a program (or use the Julia program below) which chooses 10,000 independent random numbers between 0 and 1, computes the negation of the logarithm of each number, and plots a bar graph to give the number of times that the outcome falls in each interval of length 0.1 in $[0, 10]$. On this bar graph plot a graph of the density $f(x) = e^{-x}$. How well does this density fit your graph?

```
using Gadfly

v = -log(rand(10_000))
x = 0:0.1:10
y = exp(-x)

B,H = hist(v,0:0.1:10)

plot(layer(x=x,y=y,Geom.line,Theme(default_color=colorant"red")),
     layer(x=(B[1:end-1]+B[2:end])/2,y=H/length(v)/(x[2]-x[1]),Geom.bar))
```