Solutions

MATH 520 QUIZ 2
SPRING 2017
BROWN UNIVERSITY

You have 10 minutes to answer the following questions. Solutions will be available on the course website for you to check your work.

1 Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a list of vectors in a vector space V. Describe a step-by-step procedure for pruning this list to a new, linearly independent list of vectors which has the same span as the original list. You may take for granted the ability to check whether a particular vector is in the span of some other vectors.

Work through the list starting from $\vec{v}_1$ and remove each vector which is in the span of the preceding vectors. The resulting list is linearly independent by the linear dependence lemma, and the span remains the same because each removal leaves the span the same.

2 Consider a finite-dimensional vector space V and a subspace U of V. Show that if $\dim U = \dim V$, then $U = V$.

Suppose that $U \neq V$, so there exists $v \in V$ which is not in U. Take a basis $\{\vec{u}_1, \ldots, \vec{u}_n\}$ of U, and note that $\{\vec{u}_1, \ldots, \vec{u}_n, \vec{v}\}$ is lin. ind. by the linear dependence lemma. But a lin. ind. list has length no greater than $\dim V = n$, so this is a contradiction.