

MATH 520 QUIZ 1
SPRING 2017
BROWN UNIVERSITY

Solution

You have 7 minutes to answer the following questions. Solutions will be available on the course website for you to check your work.

1 Describe in words how you can recognize from a row echelon form of the augmented matrix of a system of equations whether that system is inconsistent.

If a row echelon form of $(A | \vec{b})$ has a row of the form $(0 \ 0 \ 0 \ \dots \ 0 \ \blacksquare)$, then it is inconsistent. Otherwise, it is consistent.

2 Describe in words how to recognize from an augmented matrix in row echelon form whether the corresponding system (assuming it is consistent) has exactly one solution.

The system has exactly one solution if and only if every variable's column has a pivot position.

3 Find the reduced row echelon form of

$$\begin{bmatrix} 1 & 2 & -4 & 5 \\ 5 & -1 & 0 & 3 \\ -9 & 4 & -4 & -1 \end{bmatrix}$$

and write down the solution of the system of equations for which this is the augmented matrix.

$$\begin{bmatrix} 1 & 2 & -4 & 5 \\ 5 & -1 & 0 & 3 \\ -9 & 4 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -4 & 5 \\ 0 & -11 & 20 & -22 \\ 0 & 22 & -40 & 44 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -4 & 5 \\ 0 & -11 & 20 & -22 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 0 & -4/11 & 1 \\ 0 & -11 & 20 & -22 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -4/11 & 1 \\ 0 & 1 & -20/11 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

So x_3 is free, $x_2 = 2 + \frac{20}{11}x_3$,

$x_1 = 1 + \frac{4}{11}x_3$. So we get:

$$\left\{ \begin{pmatrix} 1 + \frac{4}{11}x_3 \\ 2 + \frac{20}{11}x_3 \\ x_3 \end{pmatrix} : x_3 \in \mathbb{R} \right\}$$

$$-4 + \frac{2}{11}(20) = -\frac{44}{11} + \frac{40}{11} = \frac{-4}{11}$$

$$5 + \frac{2}{11}(-22) = \frac{55}{11} - \frac{44}{11} = 1$$