MATH 520 PROBLEM SET 9
SPRING 2017
BROWN UNIVERSITY
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This problem set is due at the end of the day on Wednesday, the 19th of April 2017.

Problem 1

Suppose that A is an invertible matrix and that λ is an eigenvalue of A. Show that $\lambda \neq 0$ and that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

Problem 2

Suppose that A is a square matrix each of whose rows has the same sum s. Show that s is an eigenvalue of A. Hint: don't use determinants; leverage the constant row sum to search for an eigenvector directly.

Note: as an example, the matrix

$$A = \begin{bmatrix} 2 & -2 & 4 \\ 1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

has a constant row sum of 4.

Problem 3

Find the eigenvalues of the following matrix, as well as the dimension of the eigenspace of each eigenvalue.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Problem 4

Suppose that $\mathbf{v}$ is a nonzero vector in $\mathbb{R}^3$, and suppose A is a 3×3 matrix with distinct real eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$. Suppose that $|A^n \mathbf{v}|$ (the length of the vector $A^n \mathbf{v}$) converges to 0 as $n \to \infty$. Find all possible values of λ_1.

Hint: the answer will be an open interval in $\mathbb{R}$.

Problem 5

In this problem we will show that if an $n \times n$ matrix A is diagonalizable with eigenvalues $\lambda_1, \ldots, \lambda_n$, then

$$\det A = \lambda_1 \cdots \lambda_n.$$

Show this in two different ways:

(a) Write $A = PDP^{-1}$, where D is the diagonal matrix whose diagonal entries are the eigenvalues of A and P is the matrix whose columns are the eigenvectors of A. Take the determinant of both sides.

(b) Show that the determinant of A is equal to the characteristic polynomial of A evaluated at $\lambda = 0$, and show that the characteristic polynomial is equal to $(-1)^n(\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n)$. Recall from basic algebra that every polynomial $p(x)$ of degree n which has n roots factors as $p(x) = c(x - r_1) \cdots (x - r_n)$, where c is some nonzero constant and $r_1, \ldots, r_n$ are the roots of p.