MATH 520 PROBLEM SET 8
SPRING 2017
BROWN UNIVERSITY
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*This problem set is due at the end of the day on Wednesday, 5 April 2017. Please write up your solutions legibly, (**starting a new page for each problem**), scan them, and upload them using Gradescope (submission instructions on the course website). There are also MyMathLab problems due at the same time.*

1 (a) Identify three different bases of $\mathbb{P}_2$, the vector space of polynomials of degree at most 2. (b) Find a list of three polynomials which is *not* a basis of $\mathbb{P}_2$. To make this problem nontrivial, we require that this non-basis have a nonzero constant term in at least one of the polynomials, a nonzero linear term in a least one of the polynomials, and a nonzero quadratic term in at least one of the polynomials.

2 Suppose that T is a linear transformation from one vector space V to another vector space W. Suppose that Z is a subspace of W. Show that $T^{-1}(Z)$, the set of vectors in V that map to some vector in Z, is a linear subspace of V.

3 (#29 in Section 4.5) Indicate whether each statement is true or false. Justify your answers.

(a) If there exists a list $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ that spans V, then $\dim V \leq p$.

(b) If there exists a linearly independent list $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ in V, then $\dim V \geq p$.

(c) If $\dim V = p$, then there exists a spanning list of $p + 1$ vectors in V.

4 Suppose that U and V are both five dimensional subspaces of $\mathbb{R}^9$. Show that $U \cap V \neq \{\mathbf{0}\}$. Note $U \cap V$ is the set of all vectors which are in both U and V. (Hint: suppose for the sake of contradiction that $U \cap V = \{\mathbf{0}\}$, consider bases of U and V, and argue that putting these two bases together in a single list yields a linearly independent list in $\mathbb{R}^9$.)

5 Suppose that U is a nonzero subspace of V, and that T is an injective linear transformation from V to W. Show that $\dim T(U) = \dim U$. Hint: to obtain a basis for $T(U)$, start with a basis for U.