MATH 520 PROBLEM SET 5
SPRING 2017
BROWN UNIVERSITY

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*This problem set is due at the end of the day on Wednesday, 9 March 2017. Please write up your solutions legibly, (**starting a new page for each problem**), scan them, and upload them using Gradescope (submission instructions on the course website). There are also MyMathLab problems due at the same time.*

1 (§2.2, # 37) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}$. Construct a 2×3 matrix C (by trial and error) using only $-1, 1, 0$ as entries, such that CA is equal to the 2×2 identity matrix. Calculate AC to see that AC is not the 3×3 identity matrix.

2 (§2.3, #41) Using Julia or otherwise, solve both systems:

$$\begin{aligned} 4.5x_1 + 3.1x_2 &= 19.249 \\ 1.6x_1 + 1.1x_2 &= 6.843, \end{aligned}$$

and

$$\begin{aligned} 4.5x_1 + 3.1x_2 &= 19.25 \\ 1.6x_1 + 1.1x_2 &= 6.84. \end{aligned}$$

(a) If we regard the first system as the "true" system and the second as a rounded off approximation, how much percent error did we introduce in the x_1 value of the solution by rounding off? What percent error did we introduce in x_2?

(b) Calculate the determinant of the coefficient matrix of this system to see that it is very close to 0.

(c) Comment on how the solution set of the original system of equations would be different if the determinant were *actually* zero, for example if we changed the coefficient 1.1 to $(3.1)(1.6)/4.5 = 1.1020\overline{2}$. Hint: you want to approach this one conceptually, probably *not* computationally, since it'll be tricky to enter 1.10202222... into your computer exactly[1].

(d) Comment on how the percent error in the value of x_1 (resulting from the same rounding of the right-hand side we did above) would be different if the determinant were *far from* zero, for example if we changed the coefficient 1.1 to some totally different number like 3.0.

3 (§2.8 ,# 35) Suppose the columns of an $m \times n$ matrix form a basis for $\mathbb{R}^m$. What can you conclude about the relationship between m and n?

4 (§2.9, #20) What is the rank of a 4×5 matrix whose null space is three-dimensional?

5 (§2.9, #26) Suppose columns 1, 3, 5, and 6 of a matrix A are linearly independent (though not necessarily pivot columns) and A has rank 4. Explain why the columns 1, 3, 5, and 6 must be a basis for the column space of A.

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[1] Though it can be done in Julia, by entering `3.1` as the exact rational number `31//10`, etc.