MATH 520 PROBLEM SET 3
SPRING 2017
BROWN UNIVERSITY
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*This problem set is due at the end of the day on Wednesday, 15 February 2017. Please write up your solutions legibly (**starting a new page for each problem**), scan them, and upload them using Gradescope (submission instructions on the course website). There are also MyMathLab problems due at the same time.*

$\boxed{1}$ (from #33 in Section 1.4 in the book) Suppose A has 4 rows and 3 columns, and suppose $\mathbf{b} \in \mathbb{R}^4$. If $A\mathbf{x} = \mathbf{b}$ has exactly one solution, what can you say about the reduced row echelon form of A? Explain.

$\boxed{2}$ Indicate whether each statement is true or false. If it is false, give a counterexample.

(a) The vector $\mathbf{b}$ is a linear combination of the columns of A if and only if $A\mathbf{x} = \mathbf{b}$ has at least one solution.

(b) The equation $A\mathbf{x} = \mathbf{b}$ is consistent **only** if the augmented matrix $[A \quad \mathbf{b}]$ has a pivot position in each row.

(c) If matrices A and B are row equivalent $m \times n$ matrices and $\mathbf{b} \in \mathbb{R}^m$, then the equations $A\mathbf{x} = \mathbf{b}$ and $B\mathbf{x} = \mathbf{b}$ have the same solution set.

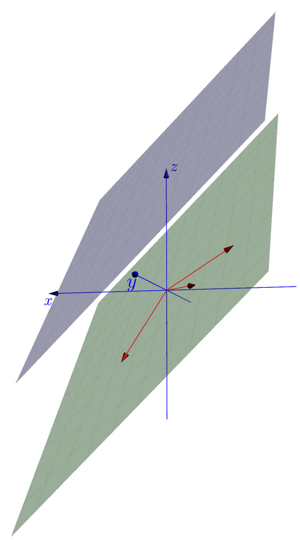
(d) If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^m$.

$\boxed{3}$ Suppose

$$A = \begin{bmatrix} -2 & 4 & -7 & -5 \\ -7 & -1 & 6 & -2 \\ -6 & 5 & 1 & 0 \\ 3 & -2 & 5 & 6 \\ 4 & -5 & -5 & -6 \end{bmatrix}.$$

Then $\mathbf{x} = (1, 1, 1, -1)$ is in the solution set of equation the $A\mathbf{x} = \mathbf{0}$ (think of this $\mathbf{x}$ as a column vector; we typeset it horizontally for convenience). Use this information to find nonzero constants c_1, c_2, c_3, c_4, such that $c_1\mathbf{v}_1 + \cdots + c_4\mathbf{v}_4 = \mathbf{0}$, where $\mathbf{v}_1, \ldots, \mathbf{v}_4$ are the columns of A.

$\boxed{4}$ Consider the vectors $\mathbf{u} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.



(a) Verify, by showing that they do not span $\mathbb{R}^3$, that these vectors are coplanar.

(b) Write down a system of equations whose solution set is equal to the span of $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ (the green plane in the figure).

(c) The blue plane in the figure is parallel to the green one and passes through the point $(0, 0, 3)$. Use what you know about systems of nonhomogeneous linear equations to alter the system you gave as an answer to (b) so that the solution set of your new system is equal to the blue plane.

$\boxed{5}$ (a) Explain in detail why the columns of a 10×13 matix span $\mathbb{R}^{10}$ if and only if every row contains a pivot position.

(b) Explain why the columns of a 10×7 matrix cannot span $\mathbb{R}^{10}$.