MATH 520 PROBLEM SET 10
SPRING 2017
BROWN UNIVERSITY
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This problem set is due at the end of the day on Wednesday, the 26th of April 2017.

Problem 1

Suppose that A is a 7×7 matrix with three distinct (real) eigenvalues. One of its eigenspaces is two-dimensional, and another eigenspace is four-dimensional. Is A diagonalizable? Explain your reasoning carefully.

Problem 2

Let A be an $n \times n$ matrix with complex entries, and let $\mathbf{v}$ be a complex eigenvector of A with eigenvalue $\lambda \in \mathbb{C}$. Show that for each nonzero complex scalar μ, the vector $\mu\mathbf{v}$ is an eigenvector of A.

Problem 3

Show that for vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, we have

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2.$$

Hint: write out the left-hand side in terms of inner products, apply the distributive property, and simplify.

Problem 4

Show that if V is a subspace of $\mathbb{R}^n$, then $V^\perp$ is a subspace of $\mathbb{R}^n$.

Problem 5

Find an orthogonal basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ of $\mathbb{R}^3$ where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$. (Hint: we will learn a general procedure for this, but you don't need it here. Trial and error works.)