MATH 520 PRACTICE MIDTERM II
SPRING 2017
BROWN UNIVERSITY
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This is a pencil-and-paper-only exam. You have two hours.

Problem 1(a)

Solve the matrix equation

$$AB\mathbf{x} + \mathbf{b} = 2AB\mathbf{x}$$

for $\mathbf{x}$, where A and B are invertible $n \times n$ matrices and $\mathbf{b}$ is an $n \times 1$ vector. Your final answer should be in terms of A, B, and $\mathbf{b}$ and should not contain parentheses.

Solution

Final answer:

Problem 1(b)

Show by substitution that the matrix $C = B^{-1}A$ satisfies the matrix equation $B^2CA^{-1} = B$.

Solution

Problem 2

The matrices

$$\begin{bmatrix} -2 & 4 & 6 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ -3 & 1 & -4 & 0 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -2 & 0 & 1 & -15 & -6 \\ 0 & 1 & 0 & -1 & 13 & 5 \\ 0 & 0 & 1 & \frac{1}{2} & -5 & -2 \end{bmatrix}$$

are row equivalent. Find

$$\begin{bmatrix} -2 & 4 & 6 \\ 1 & 0 & 2 \\ -3 & 1 & -4 \end{bmatrix}^{-1}.$$

Solution

Final answer:

Problem 3(a)

The set $\mathcal{M}_{2\times 2}$ of 2×2 matrices with real entries, equipped with matrix addition and scalar multiplication, is a vector space. The *trace* $T(A)$ of a 2×2 matrix A is defined to be the sum of its diagonal entries. In other words, the trace of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $a + d$. Show that T is a linear transformation from $\mathcal{M}_{2\times 2}$ to $\mathbb{R}^1$.

Solution

Problem 3(b)

Find the rank and the nullity of T.

Solution

Final answer:

Problem 4(a)

For which values of t is the following matrix invertible? Hint: this problem requires almost no computation; inspect the matrix carefully.

$$\begin{bmatrix} 2 & -3 & 5 & 1 & 5 & -2 \\ 1 & 1 & -5 & -3 & 0 & -5 \\ 2 & -3 & 5 & t^2 & 5 & -2 \\ -4 & -3 & -2 & 4 & -2 & -1 \\ 5 & -5 & 3 & -4 & 0 & -4 \\ -2 & -5 & 1 & 3 & -3 & 5 \end{bmatrix}$$

Solution

Final answer:

Problem 4(b)

Show that if A is a square matrix then $\det(A^T A) \geq 0$.

Solution

Problem 5

Show that $S = \{f \in C([0,1]) : f(0)f(1) \leq 0\}$ is not a linear subspace of $C([0,1])$. (In words: S is the set which contains every continuous function f from $[0,1]$ to $\mathbb{R}$ with the property that its values at 0 and at 1 have a nonpositive product.)

Solution

Problem 6

Consider the vector space $\mathbb{P}_3$ of polynomials of degree 3 or less, and consider the basis

$$\mathcal{B} = \{1, \quad 1+t, \quad 1+t+t^2, \quad 1+t+t^2+t^3\}$$

of $\mathbb{P}_3$. Find the coordinates of $-1+t^2-3t^3$ with respect to $\mathcal{B}$.

Solution

Final answer:

Problem 7(a)

Suppose that W is a ten-dimensional vector space. Suppose that U and V are subspaces of W, and that $\dim U = 8$ and $\dim V = 4$. Show that $U \cap V$ is a subspace of W.

Solution

Problem 7(b)

Show that $2 \leq \dim U \cap V \leq 4$.

Solution