MATH 520 NON-HOMEWORK 11
MATH 520
SPRING 2017

This problem set is for practice only.

Problem 1

Find an orthogonal basis for the column space of $\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$

Problem 2

Find the 5×5 matrix A such that the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is the orthogonal projection onto the line spanned by $\begin{bmatrix} 4 \\ -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$.

Problem 3

True or false:

- If W is a subspace of $\mathbb{R}^n$ and $\mathbf{v} \in W \cap W^\perp$, then $\mathbf{v} = 0$.
- If an $n \times p$ matrix U has orthonormal columns, then UU^T is the identity matrix
- If the columns of an $n \times p$ matrix U are orthonormal, then $UU^T\mathbf{y}$ is the orthogonal projection of $\mathbf{y}$ onto the column space of U.
- If $\mathbf{y} \in W$, then the orthogonal projection of $\mathbf{y}$ onto W is $\mathbf{y}$.

Problem 4

Suppose that W is a subspace of $\mathbb{R}^n$. Show that $\dim W + \dim W^\perp = n$, as follows: (i) begin with a basis of W and extend it to a basis of $\mathbb{R}^n$, and (ii) use that basis of $\mathbb{R}^n$ to come up with a orthogonal basis for W and an orthogonal basis for $W^\perp$.

Problem 5

Recall for any $m \times n$ matrix A, the row space of A is the orthogonal complement of the null space of A. Show that the linear transformation $T : \text{Row } A \to \text{Col } A$ defined by $T(\mathbf{x}) = A\mathbf{x}$ is bijective (!).