MATH 520 MIDTERM II
SPRING 2017
BROWN UNIVERSITY
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Name:

This is a pencil-and-paper-only exam. You have two hours.

Problem 1(a) (8 points)

Find $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 2 & 1 \end{bmatrix}^{-1}$

Solution

Final answer:

Problem 1(b) (8 points)

Suppose $A^{-1}\mathbf{v}_1 = \mathbf{e}_1$, $A^{-1}\mathbf{v}_2 = \mathbf{e}_2$, and $A^{-1}\mathbf{v}_3 = \mathbf{e}_3$, where $\mathbf{v}_1 = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$. Find A.

(Note: $\mathbf{e}_1\mathbf{e}_2, \mathbf{e}_3$ denote the standard basis vectors in $\mathbb{R}^3$.)

Solution

Final answer:

Problem 2(a) (8 points)

Consider the linear transformation $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x \\ 4y \end{bmatrix}$. Define T to be the 135-degree counterclockwise rotation in $\mathbb{R}^2$. Find the determinant of the composition $S \circ T$. Explain your reasoning.

Solution

Problem 2(b) (8 points)

Show that $\det(kA) = k^n \det A$, if k is a real number and A is an $n \times n$ matrix.

Solution

Final answer:

Problem 3(a) (10 points)

The matrices $A = \begin{bmatrix} 1 & 4 & 5 & 8 & 2 \\ 0 & 1 & -2 & 3 & 5 \\ -2 & -7 & -12 & -13 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 13 & -4 & -18 \\ 0 & 1 & -2 & 3 & 5 \\ 0 & 2 & -4 & 6 & 10 \end{bmatrix}$ are row equivalent.

Find a basis for the row space of A and a basis for the column space of A.

Solution

Final answer:

Problem 3(b) (5 points)

Find the rank and the nullity of A.

Solution

Final answer:

Problem 4(a) (8 points)

Consider the set $S = \{f(x) = 0$ for **some** $x \in [0,1]\}$ of continuous functions from $[0,1]$ to $\mathbb{R}$ which are zero somewhere. For example, the function $\left(x - \frac{1}{2}\right)^3$ is in S since it vanishes at $x = \frac{1}{2}$, but the function $1 + x^2$ is not in S.

Show that S **is** closed under scalar multiplication in $C([0,1])$ but is **not** closed under vector addition. So is S a subspace of $C([0,1])$?

Solution

Problem 4(b) (8 points)

Consider the subset of $\mathbb{P}_4$ consisting of all polynomials whose cubic and quadratic terms have the same coefficient. For example, $-1 + 3t^2 + 3t^3 + t^4$ is in this set, while $-1 + 2t^2 + 3t^3 + t^4$ is not. Is this set a subspace of $\mathbb{P}_4$? Explain your reasoning.

Solution

Problem 5 (10 points)

Consider the linear transformation $T : \mathbb{P}_7 \to \mathbb{P}_7$, where $T(p) = p''$. In other words, T acts by taking the second derivative. So, for example, $T(-3t^4 + t^2 - t) = -36t^2 + 2$.

Find the range and the kernel of T. Feel free to describe these sets using either a verbal description or math notation, as you prefer (as long as they are clearly specified). Explain your reasoning.

Solution

Problem 6(a) (6 points)

Show that $\{1+t^2, 1+t+2t^4\}$ is a basis of the span of $\{1+t^2, 1+t+2t^4\}$ (here $1+t^2$ and $1+t+2t^4$ are polynomials in the vector space of all polynomials, with the usual notions of addition and scalar multipliation). Find the coordinates of $1+4t-3t^2+8t^4$ with respect to the basis.

Solution

Problem 6(b) (9 points)

Use Cramer's rule to solve

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

for x and y in terms of a, b, c, d, e, f, assuming that $ad - bc \neq 0$.

Solution

Problem 7 (12 points)

Suppose that U and V are four-dimensional subspaces of a 10-dimensional vector space W. (a) Show that $U \cup V$ is not necessarily a subspace of W. (b) Show that the span of $U \cup V$, which *is* a subspace of W, is not equal to W.

Solution

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