Brown University
Problem Set 8
Instructor: Samuel S. Watson
Due: 10 November 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

Find the volume of the region W that represents the intersection of the solid cylinder $x^2 + y^2 \leq 1$ and the solid ellipsoid $2(x^2 + y^2) + z^2 \leq 10$.

Solution

Final answer:

Problem 2

Consider the region R between the parabolas $y = 1 - x^2$ and $y = x^2 - 7$. Find $\iint_R xy\, dA$.

Solution

Final answer:

Problem 3

Evaluate

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} x^2 + y^2 + z^2\, dz\, dx\, dy$$

by rewriting the integral in spherical coordinates.

Solution

Final answer:

Problem 4

Find the mass of the cylinder bounded by the surfaces $z = 0, z = 1$, and $x^2 + y^2 = 1$ whose density at (x, y, z) is given by $\rho(x, y, z) = z\sqrt{x^2 + y^2}$.

Solution

Final answer:

Problem 5

Find the region E in $\mathbb{R}^3$ for which

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)\, dV$$

is as large as possible.

Solution

Problem 6

(a) Explain why the integral of the function $f(x, y, z) = \frac{1}{x+y+z+1}$ over the cube $[0, 1] \times [0, 1] \times [0, 1]$ is equal to $\lim_{n\to\infty} S_n$, where

$$S_n = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{f(i/n, j/n, k/n)}{n^3}.$$

(b) At `sagecell.sagemath.org`, use the code [click here]

```
n = 20
var("x","y","z","i","j","k")
f(x,y,z) = 1/(x+y+z+1)
assume(0<x<1);assume(0<y<1);assume(0<z<1)
I = integrate(integrate(integrate(f(x,y,z),x,0,1),y,0,1),z,0,1)
S = sum(sum(sum(f(i/n,j/n,k/n)/n^3,k,1,n),j,1,n),i,1,n)
(I,S,N(I),N(S))
```

which gives the exact values of the integral and S_{20} followed by their decimal representations, to show that S_{20} is close to the value of the integral. Increase n (just change the first line of code to assign a different value to n) by multiples of 5 to **find the least value of** n such that n is a multiple of 5 and S_n differs from I by less than 0.01.

Solution

Final answer: