

BROWN UNIVERSITY
PROBLEM SET 11
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DUE: 6 DECEMBER 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{A}$, where

$$\mathbf{F} = \langle xy, y^2 + e^{xz^2}, \sin xy \rangle.$$

and S is the boundary of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 2$.

Solution

Final answer:

Problem 2

Suppose that D is a region in \mathbb{R}^3 bounded by a piecewise smooth surface S . Suppose that f is a differentiable function on \mathbb{R}^3 , and define $\mathbf{n} : S \rightarrow \mathbb{R}^3$ to be the outward-pointing unit normal at each point of S . Show that

$$\iiint_D \nabla f \, dV = \iint_S f \mathbf{n} \, dA. \quad (2.1)$$

Hint: begin by applying the divergence theorem to $\mathbf{F} = f\mathbf{c}$, where \mathbf{c} is a constant vector.

Note: we define the integral of a vector-valued function to be the vector of integrals of its components.

Solution

Problem 3

Define S to be the part of the sphere $x^2 + y^2 + z^2 = 4$ which lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane. Calculate the flow of $\nabla \times \mathbf{F}$ upward through S , where $\mathbf{F} = \langle xz, yz, xy \rangle$.

Solution

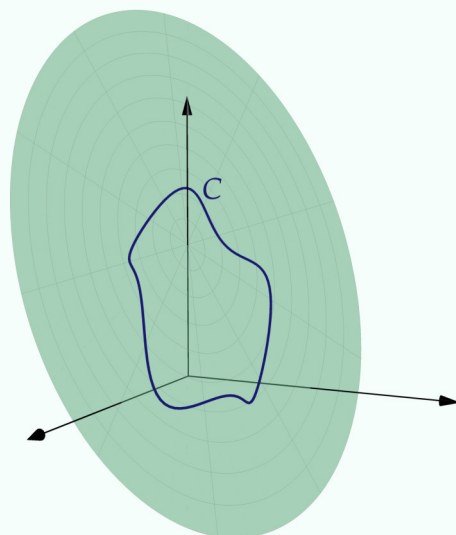
Final answer:

Problem 4

Let C be a simple closed smooth curve in the plane $x + y + z = 1$. Show that the line integral

$$\int_C z \, dx - 2x \, dy + 3y \, dz$$

depends only on the area of the region enclosed by C and not on its shape or location in the plane.



Solution

Additional space