BROWN UNIVERSITY
PROBLEM SET 10
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DUE: 1 DECEMBER 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

Sketch the vector fields $\mathbf{F}_1(x, y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{\sqrt{x^2 + y^2}}$ and $\mathbf{F}_2(x, y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{x^2 + y^2}$.

Solution

Problem 2

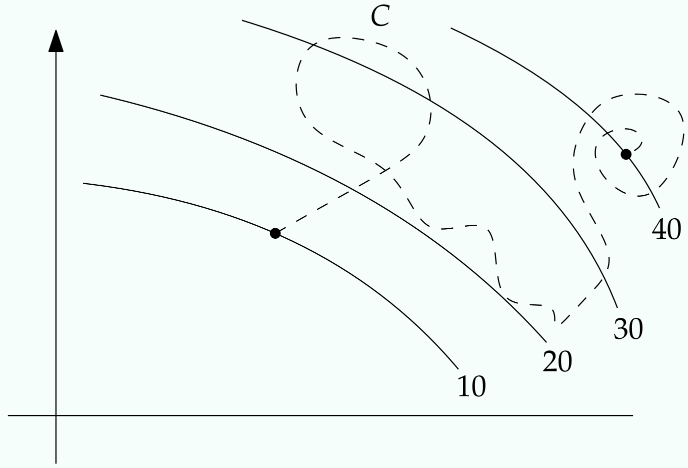
Find $\int_C (x + 2y)\,dx + x^2\,dy$ where C is the concatenation of the line segment from $(0, 0)$ to $(2, 1)$ and the line segment from $(2, 1)$ to $(3, 0)$. (Note: the notation $(x + 2y)\,dx + x^2\,dy$ is another way of writing $\mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle x + 2y, x^2 \rangle$.)

Solution

Final answer:

Problem 3

Some of the contour lines of a function $f(x, y)$ are shown in the figure below. Find $\int_C \nabla f \cdot d\mathbf{r}$, where C is the curve shown dashed starting at the left point and ending at the right point.



Solution

Final answer:

Problem 4

Find $\int_C 2xe^{-y}\,dx + (2y - x^2e^{-y})\,dy$ where C is any path from $(1, 0)$ to $(2, 1)$. Make sure to explain why your answer is correct regardless of which path C you choose.

Solution

Final answer:

Problem 5

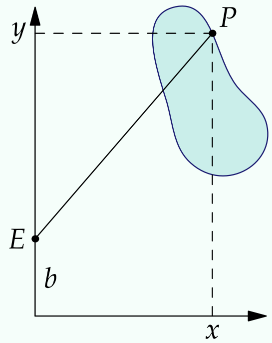
Use Green's theorem to find the line integral of $\mathbf{F} = \langle \sqrt{x^2+1}, \arctan x \rangle$ along a counterclockwise traversal of the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$.

Solution

Final answer:

Problem 6

A planimeter is a device used to calculate the area of a two-dimensional region. In this problem, we explore the mathematics behind how the planimeter works. (Thanks to Wikipedia for the picture and the description below).



The pointer P at one end of the planimeter follows the contour C of the surface S to be measured. For the linear planimeter the movement of the "elbow" E is restricted to the y-axis. Connected to the arm PE is the measuring wheel with its axis of rotation parallel to PE. A movement of the arm PE can be decomposed into a movement perpendicular to PE, causing the measuring wheel to rotate, and a movement parallel to PE, causing the measuring wheel to skid, with no contribution to its reading. You may assume that the length of PE is 1 unit.

Use Green's theorem to explain why the final reading on the measuring wheel is equal to the area of the surface S.

Hints: (i) define $b(x, y)$ to be the y-coordinate of the point E when the needle is at $P = (x, y)$. Then (ii) find the component of $\langle \Delta x, \Delta y \rangle$ which is perpendicular to $\overrightarrow{EP}$ and use your result to set up a line integral whose value equals to final reading on the meter. Then (iii) show that that the value of line integral is equal to the desired area. Hint: when it comes time to find a partial derivative of $b(x, y)$, you will be able to do this *geometrically*, without even finding a formula for b!

Solution

Problem 7

Find the flow (from the inside to the outside) of $\mathbf{F}(x, y, z) = \langle xy, y, xz \rangle$ through the cube $[0, 1]^3$.

Solution

Final answer: