Brown University
Problem Set 1
Instructor: Samuel S. Watson
Due: 15 September 2017

Print out these pages, including the additional space at the end, and complete the problems by hand. Then use Gradescope to scan and upload the entire packet by 18:00 on the due date.

Problem 1

Without using determinants, show that the range the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (2x + y, -4x - 2y)$ is a line in $\mathbb{R}^2$. (Note: this requires that you identify the line, show that every point on that line is the image of some point (x, y), and show that every point (x, y) maps to that line.)

Solution

Problem 2

Consider the line ℓ passing through $(1, -2, 0)$ and running parallel to the z-axis. Define $f(x, y, z)$ to be the distance from (x, y, z) to the line ℓ. Find a simple formula for f.

Solution

Problem 3

Find an equation of the sphere with center $(-3, 2, 5)$ and radius 4. What is the intersection of this sphere with the yz-plane?

Solution

Problem 4

Sketch or describe a solid with the property that its shadows on the three coordinate planes are a circle, an isosceles triangle, and a square, respectively. (Note: the *shadow* of a solid S in $\mathbb{R}^3$ on the xy-plane is defined to be the set of all pairs $(x, y, 0)$ such that there is some $z \in \mathbb{R}$ so that $(x, y, z) \in S$. In other words, its the set of points you get when you smash the solid directly onto the xy-plane. And similarly for the other two coordinate planes.)

Solution

Additional space