BROWN UNIVERSITY

Math 0200 Exam I

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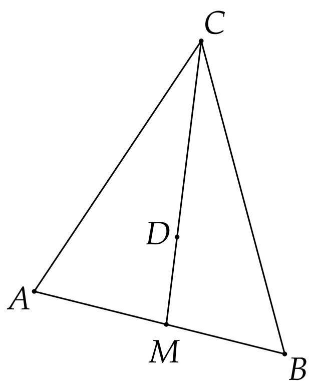
1. (10 points) Find the point on the plane $5x - z = 11$ which is closest to the point $(1, 1, 0)$.

2. A *median* of a triangle is a line segment connecting a midpoint of one of the sides with the opposite vertex. In this problem, we will use vector addition, subtract, and multiplication to show that there is a point, called the *centroid*, which lies on all three medians. Consider a triangle with vertices A, B, and C, and define $\mathbf{v} = \overrightarrow{AB}$ and $\mathbf{w} = \overrightarrow{AC}$.

Let's call D the point which is two-thirds of the way from C to the midpoint of $\overline{AB}$. In other words, D is on the line through C and the midpoint M of $\overline{AB}$, and it's twice as far from C as it is from M.



(a) (3 points) Write a simple expression for $\overrightarrow{CD}$ in terms of the vector $\overrightarrow{CM}$.

(b) (3 points) Express $\overrightarrow{CM}$ in terms of $\mathbf{v}$ and $\mathbf{w}$.

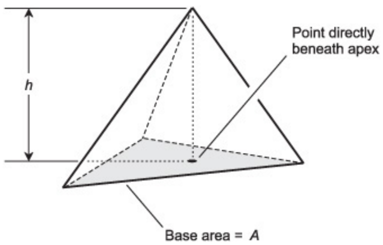
(c) (5 points) Using (a) and (b), express $\overrightarrow{AD}$ in terms of $\mathbf{v}$ and $\mathbf{w}$.

(b) (2 points) Let's call E the point which is two-thirds of the way from B to the midpoint of $\overline{AC}$. Express $\overrightarrow{AE}$ in terms of $\mathbf{v}$ and $\mathbf{w}$.

(c) (2 points) Let's call F the point which is two-thirds of the way from A to the midpoint of $\overline{BC}$. Express $\overrightarrow{AF}$ in terms of $\mathbf{v}$ and $\mathbf{w}$.

(You will see that $\overrightarrow{AD} = \overrightarrow{AE} = \overrightarrow{AF}$. This shows that $D = E = F$.)

3. Recall that the volume of a tetrahedron is equal to $1/3$ times the area A of its base times its height h (see the figure below).



The four planes $x+y=1$, $x+z=1$, $y+z=1$, and $x+y+z=1$ divide $\mathbb{R}^3$ up into several regions, one of which is a tetrahedron T.

(a) (6 points) Find the vertices of T.

(b) (6 points) Find the area of any one the faces of T.

(c) (8 points) Find the distance from the face you found in part (b) to the vertex which is not contained in that face. Use your answer and the answer to (b) to find the volume of T.

4. A particle moves in space so that its location after t seconds is given by the position vector

$$\mathbf{r}(t) = \langle t\cos(2\pi t/3), t\sin(2\pi t/3), t\rangle$$

until the time when particle reaches the sphere $x^2 + y^2 + z^2 = 2$. After that time, the particle moves in a straight line, directly away from the origin at a rate of one unit per second.

(a) (6 points) Find the coordinates of the particle's location at time $t = 5$.

(b) (6 points) Find the particle's velocity at time $t = 3/4$.

(c) (6 points) Find the arclength of the particle's path over the period from $t = 0$ to $t = 5$.

5. Consider the function f defined by

$$f(x,y) = \begin{cases} x^2 & \text{if } y = 0 \\ y^2 & \text{if } x = 0 \\ 1 & \text{otherwise.} \end{cases}$$

(a) (8 points) Explain carefully why $\dfrac{\partial f}{\partial x}(0,0) = 0$ and $\dfrac{\partial f}{\partial y}(0,0) = 0$.

(b) (8 points) Explain why f is not differentiable at $(x,y) = (0,0)$. (Hint: is f continuous at $(0,0)$?)

6. (8 points) Find

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\left[x\left(e^{\sqrt{\ln y}+y^{10}e^{y}-11\sin(\sqrt{y\tan y})}+x^{6}\right)\right]$$

7. Suppose that f is a function from $\mathbb{R}^2$ to $\mathbb{R}$, some of whose values are shown in the table below.

		x						
		−3	−2	−1	0	1	2	3
	3	36	31	28	27	28	31	36
	2	17	12	9	8	9	12	17
	1	10	5	2	1	2	5	10
y	0	9	4	1	0	1	4	9
	−1	8	3	0	−1	0	3	8
	−2	1	−4	−7	−8	−7	−4	1
	−3	−18	−23	−26	−27	−26	−23	−18

(a) (4 points) Estimate $f_x(1.5, 3)$.

(b) (4 points) Determine (with explanation) the sign of $f_{yy}(-2, 1)$.

(c) (5 points) In fact, for the function f used to generate the above table, there exists a constant c such that $f_{xy}(x, y) = c$ for all $(x, y) \in \mathbb{R}^2$. Find c, and explain your reasoning.