

Name: _____

MATH 19 QUIZ
10 NOVEMBER 2016
BROWN UNIVERSITY
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1. Classify as conditionally convergent, absolutely convergent, or divergent: $\sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{n^2}$.

Solution. The series is absolutely convergent by the comparison test, since

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n \sin n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{|\sin n|}{n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty,$$

Note that the last step holds by the integral comparison test.

2. Find the quadratic approximation of $f(x) = \ln(1+x)$ centered at $x = 0$.

Solution. The zeroth, first, and second derivatives of f at $x = 0$ are $\ln(1) = 0$, $\frac{1}{1+x} = 1$, and $\frac{-1}{(1+x)^2} = -1$, respectively. So the second-order Taylor approximation is

$$0 + 1(x - 0) + \frac{-1}{2}(x - 0)^2 = \boxed{x - \frac{1}{2}x^2}.$$