Solutions to MATH 19 Quiz 2.

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(1) Let $f(x) = A\cos x + B\cos 2x$, then:

$$6 = A\int_0^{2\pi} \cos^2 x\,dx = A\int_0^{2\pi} \frac{1+\cos 2x}{2}\,dx = A\cdot\pi. \Rightarrow A = 6/\pi.$$

$$13 = B\int_0^{2\pi} \cos^2 2x\,dx = B\int_0^{2\pi} \frac{1+\cos 4x}{2}\,dx = B\cdot\pi \Rightarrow B = 13/\pi.$$

$\therefore$ $f(x) = \frac{6}{\pi}\cos x + \frac{13}{\pi}\cos 2x$ satisfies the requirement. #

(2) $\int \sec x \tan^3 x\,dx$.

Note that $\sec x\tan x\,dx = d(\sec x)$.

So let $u = \sec x$, then $du = \sec x\tan x\,dx$.

$$= \int \tan^2 x \cdot \sec x\tan x\,dx.$$

$$= \int \tan^2 x\,du$$

$$= \int (\sec^2 x - 1)\,du.$$

$$= \int (u^2 - 1)\,du.$$

$$= \frac{1}{3}u^3 - u + C.$$

$$= \frac{1}{3}\sec^3 x - \sec x + C.$$

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