

MATH 19 PROBLEM SET 6

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- 1 Solve the following initial value problem.

$$f'(x) = xe^{x^2 - \ln f(x)^2}$$
$$f(0) = \sqrt[3]{3}$$

- 2 Find a function f satisfying

$$f'(t) = 1/f(t)$$
$$f(1) = 3.$$

- 3 Find the general solution of the differential equation

$$g''''(t) = (g'''(t))^{1/2}.$$

Hint: begin by making a substitution to reduce to a first-order differential equation.

- 4 Newton's law of cooling says that the speed of cooling of an object at temperature T is proportional to the difference between T and the ambient temperature T_a . Derive and solve a differential equation modeling the temperature $T(t)$ of an object that begins at 100 degrees and takes 10 minutes to cool to 90 degrees in a 70 degree room. (Hint: denote by k the constant of proportionality, solve the differential equation, and use the given data to find k .)

- 5 Find the general solution of

$$f'(x) - f(x) = \sin 3x.$$

- 6 Find the general solution of

$$f''(x) + 2f'(x) + f(x) = x^2.$$

- 7 (a) Show that the area under the graph of $f(x) = \frac{1}{x^2}$ over the interval $[-1, 1]$ is infinite. (b) Show that the improper integral $\int_0^\infty \sin x \, dx$ diverges.

- 8 Show that $\int_{-\infty}^0 e^x \, dx = -\int_0^1 \ln x \, dx$ in two ways: (i) calculate both integrals, and (ii) geometrically, by graphing both integrands and shading the regions whose areas are represented by the two integrals.

- 9 Find $\int_0^\infty x^n e^{-x} \, dx$ for $n = 0, 1, 2, 3, 4$. Note: you will want to use your result for $n = 0$ to find the result for $n = 1$, and so on.

- 10 THIS EXERCISE IS VERY IMPORTANT

(a) Find which values of $p \in \mathbb{R}$ have the property that $\int_1^\infty x^p \, dx$ converges. Note: you will see that the case $p = -1$ requires separate consideration, because of division-by-zero issues.

(b) Find which values of $p \in \mathbb{R}$ have the property that $\int_0^1 x^p \, dx$ converges.

(c) Are there any values of p for which $\int_0^\infty x^p \, dx$ converges?