MATH 19 PROBLEM SET 3
FALL 2016
BROWN UNIVERSITY
SAMUEL S. WATSON

1. You are inspecting an airplane engine when someone turns it on. The propeller sucks you toward it with $F(x) = 300\left(\frac{6\text{ m}}{x}\right)$ newtons of force. How much work must you do to retreat from $x = 6$ meters to a safe distance of $x = 100$ meters?

2. (a) A 60 N person steps on a scale, and the spring is depressed x meters. Express the potential energy stored in the spring in terms of x (the spring constant k should not appear in your answer).

(b) What is the change in gravitational potential energy as the person drops x meters on the scale?

(c) (ungraded) The answers to (a) and (b) are not the same. How does this not contradict conservation energy?

3. How much work is required to pump out a swimming pool (shaped like a rectangular prism) if the area of the base is 800 square meters, the water is 4 meters deep, and and the top is one meter above water level?

4. Find the length of the graph of $\frac{1}{8}x^2 - \ln x$ over the interval $[1, 2]$.

5. Use the arclength formula to show that the circumference of a unit circle is equal to 2π.

6. Find a function F such that the graph of $r = F(\theta)$ is equal to the vertical line through the point $(3, 0)$.

7. Convert each of the following points from Cartesian to polar or vice versa.

(a) $(r, \theta) = (2, \pi/2)$

(b) $(r, \theta) = (\sqrt{20}, \pi/4)$

(c) $(r, \theta) = (3\pi, 3\pi)$

(d) $(x, y) = (\sqrt{2}, -\sqrt{2})$

(e) $(x, y) = (-4, 0)$

(f) $(x, y) = (3, 4)$

(For (f), you can leave your answer for θ as an inverse trig function of a rational number)

8. Let e be a number between 0 and 1, and consider the equation $r = \frac{1}{1 - e\cos\theta}$. Convert this equation to Cartesian coordinates and express it in the form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

(where h, k, a, and b are expressions in terms of e) to show that it represents an ellipse.

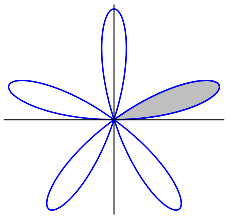

Figure for 9

9. The graph of $r = \sin(5\theta)$ is shown to the right. Find the area of the shaded region.

10. A portion of the graph of $r = \theta$ is shown below. Find the area of the dark gray region.