MATH 19 PROBLEM SET 2
FALL 2016
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1. Evaluate each of the following integrals. You can use principles from class (rather than direct calculation), but in that case you should explain in words the principle you are using.

   (a) $\int_0^{2\pi} \sin 2x \cos 3x \, dx$

   (b) $\int_0^{6\pi} \sin^2 16x \, dx$

   (c) $\int_0^{2\pi} \sin x(\cos x - 2\sin x + 3\sin 2x + 4) \, dx$

   (d) $\int_0^{2\pi} (\sin x + \cos x + \sin 2x + \cos 2x + \cdots + \sin 10x + \cos 10x)^2 \, dx$

2. (a) Give a direct argument to show $\int_0^{2\pi} \sin^2 x \, dx = \int_0^{2\pi} \cos^2 x \, dx$ (here "direct" means "without calculating the actual value of both sides"). (b) Explain why

$$\int_0^{2\pi} \sin^2 x \, dx + \int_0^{2\pi} \cos^2 x \, dx = 2\pi,$$

   again without calculating the two terms. Use (a) and (b) to show that $\int_0^{2\pi} \sin^2 x \, dx = \int_0^{2\pi} \cos^2 x \, dx = \pi$.

3. Suppose that the function $f(x) = x$ for $x \in [0, 2\pi]$ is equal to an expression of the form

$$A \sin x + B \cos x + C \sin 2x + D \cos 2x + E \sin 3x + \cdots$$

   Repeat the method illustrated in the solution of Example 2.4 from the notes to find A, B, C, and D (you may assume it's OK to distribute the integral sign across the infinite sum above).

4. Compute each of the following integrals.

   (a) $\int \cos^3 x \sin^5 x \, dx$

   (b) $\int \sin^4 x \cos^2 x \, dx$

   (c) $\int \sec x \tan^3 x \, dx$

   (d) $\int \csc^4 x \cot^2 x \, dx$

5. Use the cosine sum-angle formula to find constants C and α so that

$$\sin x + \cos x = C \cos(x + \alpha)$$

   Use this identity to find the maximum and minimum values of $\sin x + \cos x$ without using calculus. Show more generally that for any A and B, there exist D and β so that

$$A \sin x + B \cos x = D \cos(x + \beta).$$

6. Find $\displaystyle\int \frac{1}{\cos x - \sin x} \, dx$. (Hint: use the previous exercise.)

7. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{\sqrt{x^2 - 25}}{x} \, dx$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{1 - x^2}} \, dx$

(c) $\int_{-1}^{1} (1 - x^2)^{3/2}\, dx$

(d) $\int_{1}^{4} \frac{1}{\sqrt{x^2 - 1}}\, dx$

8. Trig sub $x = \sin\theta$ to show $\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin(x)$. Trig sub $x = \cos\theta$ to show $\int \frac{1}{\sqrt{1-x^2}}\, dx = -\arccos(x)$. Wait, what? Explain this apparent contradiction.

9. Find $\int \frac{1}{x^2+2x}\, dx$ by completing the square in the denominator and making a suitable substitution.

10. We learned how to integrate functions of the form $\sin^m x \cos^n x$ and functions of the form $\sec^m x \tan^n x$, where m and n are nonnegative integers. This may seem somewhat arbitrary, but actually many products of the six trig functions can be rewritten as one of these cases (or $\csc^m x \cot^n x$, which may be integrated using the same approach as for $\sec^m x \tan^n x$):

    Show that any function of the form $\sin^a x \cos^b x$, where a and b are integers which are not both negative, can be written as a sum of functions of the form $c \sin^m x \cos^n x$, $c \sec^m x \tan^n x$, or $c \csc^m x \cot^n x$, where m and n are *nonnegative* integers (and the coefficients c are constants).