

MATH 19 PROBLEM SET 1
FALL 2016
BROWN UNIVERSITY
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1. (Review) Evaluate each of the following integrals.

(a) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

(b) $\int \frac{1}{1 - e^{-x}} dx$

(c) $\int_{-1}^3 e^{|x|} dx$

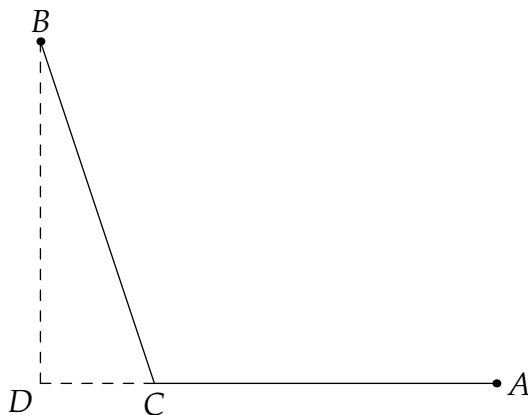
(d) $\int_0^{1/2} \frac{1}{1 - x^2} dx$

2. (Review) A dog located at point A wants to retrieve a ball on the surface of a lake located 30 meters from the shore, as shown. Also, the distance from A to the point D on the shore nearest the ball is 40 meters. The dog can run along the shore at a rate of 6 meters per second but can only swim at a rate of 3 meters per second. Let x denote the distance DC .

(a) Write down an expression in terms of x for how long it takes the dog to get from point A to point C .

(b) Write down an expression in terms of x for how long it takes the dog to get from point C to point B .

(c) For what value of x is the dog's total travel time from A to B minimized?



3. Evaluate each of the following integrals.

(a) $\int x^2 e^x dx$

(b) $\int_0^3 \ln(x^2 + 1) dx$

(c) $\int_0^1 x \arctan x dx$

(d) $\int x \sqrt{x + 3} dx$

(e) $\int \sin(\ln x) dx$

(f) $\int (\ln x)^2 dx$

(g) $\int_0^\pi e^x \cos x dx$

4. Integrating $e^x \sin x$ by assuming there is an anti-derivative of the form $f(x) = Ae^x \sin x + Be^x \cos x$, differentiating f , and solving for A and B . Is this easier than integration by parts?

5. Show that for all positive integers n , we have

$$\int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx.$$

6. Show that

$$\int f(x)g'(x) dx = f(x)g(x) - G(x)f'(x) + \int G(x)f''(x) dx,$$

where G is an anti-derivative of g .