

**MATH 19 PRACTICE MIDTERM I**  
**FALL 2016**  
**BROWN UNIVERSITY**  
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**1** Find  $\int_0^{\sqrt{44}} \left( \frac{x}{\sqrt{x^2 + 100}} - 2 \right) dx$

2 Find any function  $f(x)$  such that

$$\int_0^{2\pi} f(x) \cos 2x \, dx = \frac{1}{3}$$

$$\int_0^{2\pi} f(x) \sin 3x \, dx = 4$$

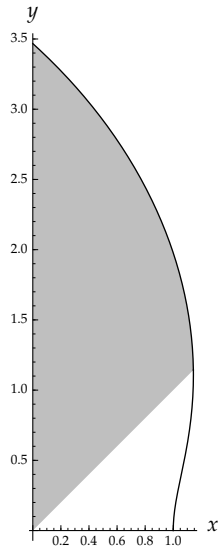
$$\int_0^{2\pi} f(x)(\sin x + \sin 2x) \, dx = 11.$$

3 Find the length of the line segment  $L$  from the origin to  $(3, 4)$  two ways:

a) by defining a function  $f(x)$  whose graph over  $[0, 3]$  is  $L$ , and using the arclength integral formula applied to  $f$ , and

(b) using Euclidean geometry.

4 The graph of the polar coordinate equation  $r = 1 + \theta^2$  over  $0 \leq \theta \leq \frac{\pi}{2}$  is shown below. Find the area of the portion of the region enclosed by this curve which lies above the line  $y = x$ . (In other words, find the area of the shaded region.)



5 Find all complex values of  $z$  satisfying the equation  $(z - i)^3 = 8$ . Express your answers in  $a + bi$  form.

6 Find a function  $f$  which satisfies all of the following equations.

$$f''(x) - f(x) = 0$$

$$f''(x) - 3f'(x) + 2f(x) = 0$$

$$f(0) = 13.$$