Name: ________________

MATH 19 PRACTICE FINAL
FALL 2016
BROWN UNIVERSITY
SAMUEL S. WATSON

1 (10 points) Find $\int \cos^2 x + \cos^{10} x \sin x + xe^x \, dx$.

2 (10 points) Consider the function

$$f(x) = \int_0^x \sqrt{4\cos^2 t - 1}\, dt.$$

Find the arclength of the graph of f over the interval $[0, \pi/2]$.

3 (10 points) Find the function f which satisfies $f(0) = 3$, $f'(0) = 6$, and

$$f''(x) + 2f(x) = 2f'(x) + e^x.$$

4 (10 points) Determine the convergence or divergence of each of the following series.

(a) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(b) $1 + \frac{1}{1+2} + \frac{1}{1+2+4} + \frac{1}{1+2+4+8} + \cdots$

5 (10 points) Determine the convergence or divergence of each of the following series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n^{n^2}}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n^2+1+2^{-n}}{n^2-10}$

6 (10 points) (a) Suppose f is a continuous function from $[0, \infty)$ to $\mathbb{R}$ and that $\int_0^\infty f(x)dx = 21$ and $\int_0^1 f(x)dx = 16$. Find

$$\lim_{b \to \infty} \int_1^b f(x)\, dx.$$

(b) Suppose that the integrals $\displaystyle\int_0^1 x^p\, dx$ and $\displaystyle\int_1^\infty x^p\, dx$ are both improper and divergent. Find p.

7 (10 points) (a) Find the real Fourier series of any 2π-periodic function $f(x)$ which is equal to 1 for all x strictly between $-\pi/2$ and $\pi/2$ and 0 for all x strictly between $\pi/2$ and $3\pi/2$.

(b) Suppose f is one of the functions described in part (a), and suppose that the Fourier series for f converges to $f(x)$ for all x. Calculate $f(0)$, $f(\pi/2)$, and $f(\pi)$.

8 (10 points) Consider a physical system which responds to a periodic stimulus $V(t)$ by behaving according to the periodic solution Q of the differential equation

$$Q''(t) + 6Q'(t) + 13Q(t) = V(t).$$

Express $V(t)$ as a real Fourier series given that

$$Q(t) = \sum_{n=-\infty}^{\infty} \frac{1}{\pi(n^2+1)} e^{int}.$$

Note: You may assume that Q is twice differentiable (it is).

9 (10 points) Consider the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n}(x-3)^n$.

(a) Find the radius of convergence of this power series.

(b) Let us define $f(x) = \sum_{n=0}^{\infty} \frac{n!}{n^n}(x-3)^n$ for all x such that the infinite series on the right-hand side converges. Calculate $f^{(4)}(3)$.

10 (10 points) (a) Writing an arbitrary complex number z as $x + iy$ where x and y are real, show that $e^z \neq 0$ for all complex numbers z.

(b) Show that for all real numbers x, we have

$$1 - \frac{x^2}{2} + \frac{x^4}{2^2 \cdot 2!} - \frac{x^6}{2^3 \cdot 3!} + \frac{x^8}{2^4 \cdot 4!} - \cdots \neq 0.$$

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