MATH 19 MAKEUP MIDTERM II
FALL 2016
BROWN UNIVERSITY
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$\boxed{1}$ Determine whether the following series converge or diverge. For each example, clearly state which convergence test or tests you are using.

(a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+1)!}$

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\tfrac{\pi}{2} - \arctan n\right)$

(c) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n + 1}$

(d) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1 + \sqrt{n}}$

$\boxed{\mathbf{2}}$ (13 points) Determine the convergence or divergence of the following improper integral. Hint: watch out for improperness *inside* the interval of convergence!

$$\int_0^\infty \frac{1}{(x-1)^2}\,dx$$

$\boxed{\mathbf{3}}$ (14 points) Find

$$\sum_{k=0}^{\infty} \frac{1}{3^k} + \sum_{k=1}^{\infty} \frac{1}{3^k} + \sum_{k=2}^{\infty} \frac{1}{3^k} + \sum_{k=3}^{\infty} \frac{1}{3^k} + \cdots .$$

$\boxed{4}$ Suppose that $(a_n)_{n=1}^{\infty}$ is a positive decreasing sequence which converges to 0. For all positive integers n, define the partial sum

$$S_n = \sum_{k=1}^{n} (-1)^{k+1} a_k.$$

(a) (7 points) Explain why $S_1, S_3, S_5, S_7, \ldots$ is a decreasing sequence and $S_2, S_4, S_6, \ldots$ is an increasing sequence.

(b) (7 points) In the above context, every odd-indexed partial sum is greater than every even-indexed partial sum. In other words, $S_k > S_j$ whenever k is odd and j is even (you are not being asked to show this; just take it as given). What theorem, combined with this fact and part (a), allows you to conclude that both $S_1, S_3, S_5, S_7, \ldots$ and $S_2, S_4, S_6, \ldots$ converge as $n \to \infty$?

$\boxed{\mathbf{5}}$ (12 points) Find a function f which satisfies $f'(x) = xe^x f(x)^2$.

$\boxed{\mathbf{6}}$ (15 points) By directly calculating derivatives of f and substituting into the formula for a Taylor polynomial, find the third-order Taylor polynomial for $f(x) = \sin x + \cos 2x$ centered at $x = 0$.