

MATH 19 EXTRA FOURIER DE PRACTICE PROBLEMS
FALL 2016
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Exercise 1

See <http://www.math.brown.edu/~pflueger/math19/pset9.pdf>, Question 9. A solution is available on Professor Pflueger's website.

Exercise 2

Suppose f is a function whose complex Fourier coefficients are given by $d_n = \frac{1}{\pi n^4}$ for $n \neq 0$ and $d_0 = \frac{1}{2}$. Find the periodic solution g of the differential equation

$$g''(x) + 2g'(x) + 2g(x) = f'(x) + f(x).$$

Express your answer as a real Fourier series.

Exercise 3

Consider a spring with spring constant k attached to an object of mass m and subject to resistive force proportional to the velocity of the object (with constant of proportionality c) and subject to an external, time-varying, 2π -periodic force $F(t)$.

(a) Write a differential equation for the position $x(t)$ of the object.

(b) Let $x(t)$ be the periodic solution to the DE in (a). Find the most resonant frequency of this system. In other words, find the integer n for which the ratio of the coefficient of e^{int} in the Fourier series for x to the coefficient of e^{int} in the Fourier series for F is as large as possible. You may express your answer in the somewhat imprecise form "one of the two integers closest to _____", where the blank is filled in with some expression involving c , k , and m .