MATH 0180

Recitation 7. (Oct, 27, 2016)

Solutions

1. $\sum_{n=1}^{\infty} (\arctan(n+1) - \arctan n)$

$= \lim_{N\to+\infty} \sum_{n=1}^{N} (\arctan(n+1) - \arctan n)$

$= \lim_{N\to+\infty} (\arctan(N+1) - \arctan 1)$

$= \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$

2. The sum equals to

$0.9 \times 1 + 0.9 \times 0.1 + 0.9 \times 0.1^2 + \cdots$

$= \lim_{N\to+\infty} \frac{0.9 \times 0.1^N - 0.9}{0.1 - 1} = \frac{0.9}{0.9} = \boxed{1}$

3. The total time equals

$t = \sqrt{\frac{2\times 10}{g}} + \sqrt{\frac{2\times 10 p}{g}} \times 2 + \sqrt{\frac{2\times 10 p^2}{g}} \times 2 + \cdots$

$= -\sqrt{\frac{20}{g}} + 2\sqrt{\frac{20}{g}} \left(1 + p^{\frac{1}{2}} + p + p^{\frac{3}{2}} + \cdots\right)$

$= \sqrt{\frac{20}{g}} \frac{2}{1 - p^{\frac{1}{2}}} - \sqrt{\frac{20}{g}} = \boxed{\sqrt{\frac{20}{g}} \cdot \frac{1+\sqrt{p}}{1-\sqrt{p}}}$

4. Use the integral test, consider

$\int_{x=2}^{+\infty} \frac{1}{x \ln x \ln\ln x} dx = \int_{x=2}^{\infty} \frac{1}{\ln x \ln\ln x} d\ln x$

$= \int_{x=2}^{\infty} \frac{1}{\ln\ln x} d\ln\ln x$

$= \int_{x=2}^{\infty} \frac{1}{u} du$ $\qquad u = \ln\ln x$, $x = e^{e^u}$

$= \ln u \Big|_{u=\ln\ln 2}^{\infty} = +\infty$

$\Rightarrow$ The sum $\boxed{\text{diverges}}$