DATA 1010
In-class exercises
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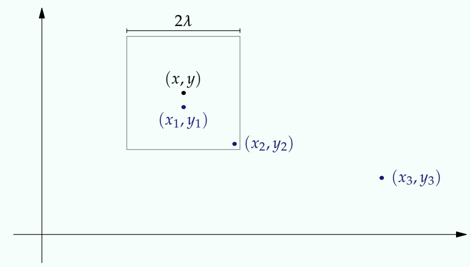
Problem 1

Put the values $K_\lambda(x_1 - x, y_1 - y), K_\lambda(x_2 - x, y_2 - y), K_\lambda(x_3 - x, y_3 - y)$ in order from least to greatest.

Recall that $K_\lambda(x, y) = D_\lambda(x)D_\lambda(y)$ where $D_\lambda(x) = \frac{70}{81\lambda}\left(1 - \frac{|u|^3}{\lambda^3}\right)^3$.



Solution

We have $K_\lambda(x_3 - x, y_3 - y) = 0$, since (x_3, y_3) lies outside the support of the kernel centered at (x, y). Next, $K_\lambda(x_3 - x, y_3 - y)$ is positive but quite small, and $K_\lambda(x_1 - x, y_1 - y)$ is the largest.

Problem 2

Suppose we have a probability density function f on a rectangle in $\mathbb{R}^2$, and we compute its values on a fine-mesh grid of points along a vertical line at position x through the rectangle and store those values in a vector `v`. Suppose that the y-coordinates of the grid points are stored in a vector `ys`.

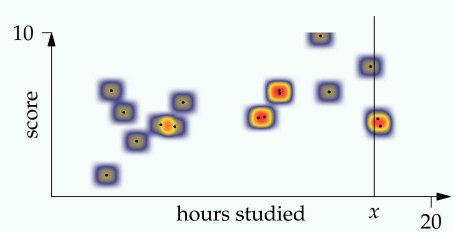
Write a line of Julia code to approximate the conditional expectation of Y given $X = x$, if (X, Y) has PDF f.

Solution

The expectation is a probability-weighted average, so we calculate an average weighted by the probabilities: `v⋅ys/sum(v)`.

Problem 3

How many of the samples in the figure shown have a nonzero contribution to the integral representing the conditional expectation of Y given $X = x$? (The heatmap shows the joint density of X and Y.)



Solution

The line passes through **three** squares. Two of the squares overlap, but all three samples contribute to the conditional expectation integral.