DATA 1010
In-class exercises
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Problem 1

Suppose that $\mathbf{X}$ is a random vector with mean zero and covariance matrix Σ. What is the covariance matrix of $A\mathbf{X}$, where A is a square matrix?

Solution

We have

$$\mathbb{E}[A\mathbf{X}(A\mathbf{X})'] = \mathbb{E}[A\mathbf{X}\mathbf{X}'A'] = A\mathbb{E}[\mathbf{X}\mathbf{X}']A' = \boxed{A\Sigma A'}.$$

Problem 2

Suppose that X is Bernoulli with $p = 0.8$ and that Y is independent of X and normal with mean zero and variance 10^{-3}. Describe the PDF of $X + Y$.

Solution

Since $X + Y$ is very close to either 0 or 1 with high probability, its mass will be concentrated near those two locations. Furthermore, nearly 80% of the mass will be right around 1, and nearly 20% will be right around 0. So we'll see a sharp hump around 0 and another sharp hump (four times taller) around 1.